A STUDY OF THE IMPACT OF RISK TOLERANCE ON MULTI-LEVEL R & D DECISION PROCESSES

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A STUDY OF THE IMPACT OF RISK TOLERANCE

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SUMMARY

This research is designed to gain insights into multi-level decision processes. It is exploratory in nature and specifically looks into the effects of the risk tolerance of decision makers on this multi-level decision process.

This process is examined in an R & D environment. The methodology used is simulation. The model used for the research is a network representation of a large hierarchical governmental organization. The model is a funding model, hence the decision investigated is the selection and funding of R & D tasks. As used in this work, this model is solved more than one time in a sequential manner. This gives the model the multi-level property needed for the research.

The research acknowledges that the multi-level decision process is an extremely complex process. However, a greater understanding of this process is possible including two interesting insights. Tentative results indicate that a risk intolerant decision maker will find fewer tasks than will his risk tolerant counterpart. More importantly, it appears that the imposition of budgetary constraints by a higher level on a lower level of the organization need not necessarily have the effect of forcing the lower level to behave in a manner more consistent with that of the higher level.

While the insights reached in this work are tentative, they do point to specific areas where further research is needed.

CHAPTER I

INTRODUCTION

Background

A hierarchical or multi-level organization is extremely complex. The decision processes occurring within such an organization are also complex. Traditionally, management scientists have taken two avenues of approach when studying decision processes in multi-level organizations. One approach has been to study individual decision making and decision processes [12] and the other has been to study the organizational and behavioral aspects of the system [24,25,33]. Each of these approaches has led to a relatively separate and well-defined body of knowledge. However, these two approaches and the separate bodies of knowledge resulting from their use, have not begun to consider multi-level decision processes in depth.

The multi-level decision process and the effects of some of the decision variables on the outcomes of this process are the subjects of this study. This will involve the use of elements from both decision theory and organization theory. The environment used for this study will be that of a research and development (R & D) organization. This environment was chosen because such organizations and the decision processes which occur therein have been relatively well researched and are relatively well understood compared to other organizational environments.

Management scientists have in the recent past devoted much time and thought to studying, hypothesizing, and modeling R & D decisions [1,3,6, 37]. This definition and understanding of the R & D decision process and the fact that many R & D decisions occur within the framework of a multi-level organization make this environment an ideal one in which to work and the decision processes excellent ones to study.

General Statement of the Research Question

The object of this thesis is to study the impact of a decision maker's risk tolerance on the multi-level R & D decision process in hierarchical organizations. The primary R & D decision to be studied will be that of project selection and allocation of a scarce resource to those selected projects. The scarce resource considered in this study is budgetary dollars. In effect, this makes the decision process under study a capital budgeting decision process. This particular decision process allows one to evaluate the effects of a decision maker's risk tolerance on that process. In this study, risk tolerance will be defined as a decision maker's propensity toward the acceptance of taking risks (risk seeking or risk tolerance) or his avoiding the taking of risks (risk averse or risk intolerance).

Due to the nature of the mathematical model used, a constrained optimization model which is solved by the out-of-kilter algorithm of network theory, multi-level decision processes can also be studied. The model and the type of organization it represents is fully described in Chapter III. A multi-level decision process is a decision process in which individuals from more than one level of an hierarchical organization participate. In such a decision process, no single individual can be pinpointed as the decision maker. This study will be concerned only with the two-level decision process. This allows the study of the interplay of the risk tolerances of at least two individuals in the process and the effects of these risk tolerances on the outcomes of the process. Another objective of this research will be to study the effectiveness of budgetary constraints in achieving resolution of differences between individuals operating with different risk tolerances within a given decision process.

All of these goals can be accomplished by using expected utility as the decision criterion in the decision process. The advantage gained from this is that it allows direct manipulation of a "risk factor" in each decision maker's value function. Moreover, when making decisions under uncertainty, many decision theorists advocate the use of expected utility as the decision criterion [13,20,23,26,38]. These experts contend that the use of expected utility most fully recognizes the risk involved in the decision (riskiness of the task) and by the use of a probabilistic weighting of the utilities, expresses a range of possible outcomes in terms of a single outcome.

Assumptions

During the performance of this research, it became necessary to make certain assumptions. Sometimes these assumptions were made in order to simplify numerical computations. Other times they were made in order to explain certain behavior which could not be mathematically modeled. Below is a list of the critical assumptions made during this research.

1. All individuals involved in the decision process use expected utility as their value criterion and each decision maker's utility function remains constant for all decisions.

2. The utility function of each decision maker is quadratic in nature.

3. Each decision maker operates with some level of risk tolerance inherent to his decision process and this level of risk tolerance remains constant for all decisions.

4. Expected utilities are additive.

5. All tasks and projects used are independent of one another.

6. At increased funding levels, task means will not be decreased and task variances will increase or decrease randomly.

7. The variances of tasks will be uniformly distributed between two points which will be arbitrarily chosen.

Other assumptions arise due to the particular model selected and are detailed in Chapter III. In summary, these assumptions are:

8. Benefit functions are piece-wise linear.

9. The benefit for an intermediate funding point is given by the piece-wise linear curve connecting the benefit points associated with the alternative funding levels.

10. A task is funded wholly within only one branch and is associated with only one project.

11. The solution procedure for the model is near-optimal, it does not always provide optimal solutions.

12. The constrained optimization model is a reasonable representation of an individual decision maker.

CHAPTER II

LITERATURE SURVEY

This thesis draws upon three distinct areas of the literature for its background. As there have been relatively few works in the literature which deal with the problem as approached by this study, the literature cited will serve to form the background and theoretical basis for the mathematical model used.

Those works currently in the literature which do deal with multi-person decisions are only partially applicable to this work. The works of Ruefli [30,31] take a decomposition...goal programming approach to the generalized multi-person decision problem. Mesarovic, Macko, and Takahara [27] attempt to deal with this problem in its most general form by the use of systems theory. None of these works is directly applicable to this study because of their general nature. The three works cited, in the most general way, explain some of the processes which occur in the multi-level decision problem. However, due to their generality they provide no directly applicable results. Also the approaches outlined in these works make no provisions for allowing the peculiarities of the R & D decision to be included therein.

R & D Project Selection

The area of R & D project selection has been investigated by a number of researchers in the recent past. Baker and Pound [3], Baker and Freeland [6], Souder [37], Alboosta and Holzman [1], Cetron, Martino, and Roepcke [8] present a comprehensive overview of the literature in this area. In particular, the Baker-Freeland paper covers some 237 articles and books. A few of the more relevant sources found in these overviews will be dealt with in greater detail.

Cramer and Smith [10] provide an interesting approach to R & D project selection. They begin by viewing the project selection process as a risky decision. From there, they advance to a utility approach to value measurement and then combine the two to form an expected utility objective function for the decision makers. The model used in this thesis makes direct use of the decision maker's objective function as advocated by Cramer and Smith.

In the area of project selection itself, many approaches can be found in the literature. Freeman [18] advocates a stochastic model to determine the size of an R & D budget and its allocation among projects. He uses probability distributions to describe projects and maximizes expected value of the resulting portfolio. Hess [21] presents a dynamic programming approach to the selection and budgeting of R & D projects. Given estimates of the discounted value of future projects as of several points in time and estimates of the probabilities of success, the model selects that set of projects which maximized the total expected value of all projects. Rosen and Souder [29] extended Hess's model by considering different optimization criteria for obtaining future finding patterns. They included expected profit, expected research successes, and return on expenditure in the criteria utilized. Their model determines: (a) the maximum expected profit from all projects over all time periods; (b) the optimal budget allocated among the projects for the current time period; (c) the optimal future expenditures for each time period; and, (d) the expected return of each project corresponding to the optimal future expenditures.

Atkinson and Bobis [2] formulated and solved a resource allocation model which explicitly accounts for the variation of allocated resources over time. The model is based upon the single criterion of expected profit. However, this model is extremely complex and would be unwieldy to implement as an aid to decision makers. Alboosta and Holzman [1] present a model for optimal funding of an R & D portfolio. The criterion Alboosta and Holzman choose for optimization is a subjective expected value in a goal oriented model. The most interesting idea presented is the formation of a function to describe project risk and this is then utilized in solving for the optimal funding pattern. Watters [39] in his Ph.D. dissertation presents yet another project selection model. This model uses a utility theory approach and treats the decision as one made under uncertainty. Watters' criteria for optimization include maximization of expected utility and minimization of risk to the organization. The model includes the capabilities of handling project dependences and probabilistic budget constraints.

Baker, Shumway, Souder, and Maher [4] present a network formulation for the selection of and resource allocation to R & D projects in a large multi-level organization. This model uses subjective three point estimates of value for each project and then solves a minimization of negative value problems. The model will not handle project dependencies but is computationally relatively simple. Hence it can be implemented in a real time mode and be of use to the decision maker. This model will be discussed in greater detail in the next chapter as it forms the basis for the research contained in this thesis.

Decision Theory

Individual decision theory is an area which has been heavily researched by both management scientists and psychologists. Just recently, an excellent review of the literature in this area was published by Slovic and Lichtenstein [36]. In this paper, the authors contrast the basic approaches to this area and detail the models formed by each approach. They run the gamut from inference in uncertain environments [7], through subjective probabilities [11], to computer simulation of thought [28]. Since this review of the literature is an excellent one, it will not be detailed here. The reader is referred to the original work for more information. However, the utility theory aspect of decision making will be dealt with in more detail here.

The reasons for turning to utility theory in the R & D decision have been succinctly stated by Watters:

> When outcomes are relatively large and risky, a preference structure is desired which reflects more than just predilection for maximum monetary return. The construction of such preference structures for use in decision-making situations falls quite naturally into the realm of utility theory [39].

A method for quantifying preference structures such that decisions of the above type can be made on a rational and internally consistent basis has been developed by von-Neumann and Morgenstern [38]. They have shown that, once a person's utility function is established for a decision situation in conformity with certain consistency axioms, the proper criterion to be used as the basis for making the decision is maximization of expected utility. Detailed discussions of the von-Neumann-Morgenstern utility theory are given by Savage [32] and implications and interpretations of the axioms appear in many sources, two of which are Luce and Raiffa [23] and Fishburn [14].

In order to make use of this theory of utility in the project selection process, some functional relationships other than discrete points read from a utility curve must be established. Markowitz [26] establishes an expectation-variance criterion function, for use in selecting efficient investment portfolios. Watters [39] on pp. 30-32 of his dissertation shows that this expectation-variance criterion function is a very close approximation to a bonafide utility function. He also shows that the quadratic utility function can also be approximated using the expectation-variance criterion and that these approximations do offer improvements over the expected return criterion in making high risk outcome decisions. Moreover, when no risk aversion is found in the decision maker, the expectation-variance criterion function degenerates to the expected return criterion.

Organization Theory

The literature of the field of organization theory is quite large and diverse. Works by authors such as Simon [35], March and Simon [25], Edwards [11], and others tend to give an excellent overview of the field. Other authors, such as Schull, Delbecq, and Cummings [33] have looked into the area they call "organizational decision making." In this work the authors study decision making and decision processes in the context of the type of organization in which they occur. However, no organizational literature was found which directly supported this particular research.

Summary

In this chapter, a brief review of the literatures relating to this research was presented. This survey touched upon three separate literature areas: decision theory, R & D project selection methodology, and organization theory. Excellent survey papers were found and presented in the areas of project selection methodology and decision theory. The area of organization theory was found to be somewhat empty, so far as direct application to this research is concerned.

CHAPTER III

DEVELOPMENT OF THE MODEL

Description of the Organization Modeled

The following is a description of the organization taken from the original paper by Baker, Shumway, Souder, Maher, and Rubenstein [5] in which the model was originally presented. This discussion is included so that the reader can get a feel for the type of organization for which this research may be relevant.

> The organization within which this study was conducted is typical of many large federal R & D organizations. It is characterized by several administrative levels, each of which has, within imposed limitations, considerable autonomy over its own operation. The budget process is sequential in nature; that is, budget guidance in the form of recommended funding is issued from each superordinate level to its immediate subordinate levels, based, in turn, on the guidance it has received and on its decision as to how the budget should be further apportioned. Thus, guidance information flows from the highest administrative level, through all intermediate levels, and on to the lowest organizational unit. In addition to budget guidance according to organizational entity, guidance is also issued according to technical areas. For example, a laboratory will receive guidance regarding its total budget and guidance indicating acceptable budgets for selected projects and groups of projects. The laboratory, in turn, will issue guidance for its subordinate organizations and for its project sub-entities.
>
> After guidance reaches the lowest organizational level, the information flow is reversed. Each subordinate level transmits a proposed budget allocation to its immediate superordinate level in which the subordinate level details how it would allocate the guidance budgets if they were in fact to be authorized. These proposed allocations are integrated at each level and are then communicated to the next higher level. This downward-upward flow cycle may recur many times for more than one set of figures. Ultimately the highest organizational level receives a proposed

budget allocation either consistent with the figures it originally issued as guidance or otherwise acceptable to it.

Eventually, the highest administrative level determines the total amount of funding which will be appropriated for the entire research organization. The appropriated allocations then flow through the organizational hierarchy in a manner analogous to the flow of the guidance information. At this point, each organizational level knows, within limits, the level of funding it can anticipate during the fiscal year and the budgetary constraints which have been imposed on its operation. Specific fiscal year budget plans are then made. Frequently, these plans must be revised during the year, since the eventual authorizations may deviate from the appropriations. Accordingly, several times during the year each organizational level is faced with a resource allocation decision which is characterized by a large number of budgetary constraints, defined both by organizational entity and by research area.

The preceding discussion is taken directly from "A Budget Allocation Model for Large Hierarchical R & D Organizations" by N. R. Baker, W. E. Souder, C. R. Shumway, P. N. Maher, and A. H. Rubenstein [5].

The Model

The model to be used in this study must possess certain characteristics. It must also allow the study of two or more individuals from different levels of the organization acting within a given decision process. This will allow the multi-level aspect of the decision process to be studied. Fortunately, among the large number of R & D models developed, there does exist one which fits the requirements of this study with very few modifications. This is the model developed and used by Baker, Maher, Shumway, Souder, and Rubenstein [5]. This model will be briefly discussed in this chapter and the necessary modifications will also be noted.

This model is designed to represent a large decentralized research organization. This organization has two distinct hierarchies which can be identified; an organizational or administrative hierarchy, and an activity hierarchy. The administrative hierarchy can be considered as consisting of a headquarters containing divisions and branches, and a subordinate level, consisting of laboratories and directories. This hierarchy is represented in Figure 1.

In the administrative hierarchy, the divisions and branches are primarily involved in planning and budget specifications. The laboratories and directories are concerned with the expenditure of budgets on specific activities. These two facets of the administrative hierarchy are linked by the tasks.

The activity hierarchy consists of the specific research activities, of which there are three. These are research programs which contain research projects or areas. The projects are composed of research tasks which are the actual entities which are performed. This activity hierarchy is represented in Figure 2.

With one assumption, a key relationship can be constructed between these two hierarchies. This relationship then allows a network representation of the research organization under consideration to be constructed. This relationship is that any task is funded wholly within a given branch and is uniquely associated with one and only one project. This ties the two hierarchies together in such a manner as to allow construction of the network shown in Figure 3.

Because of the nature of both the administrative and research hierarchies, the model can be formulated as minimal cost network flow

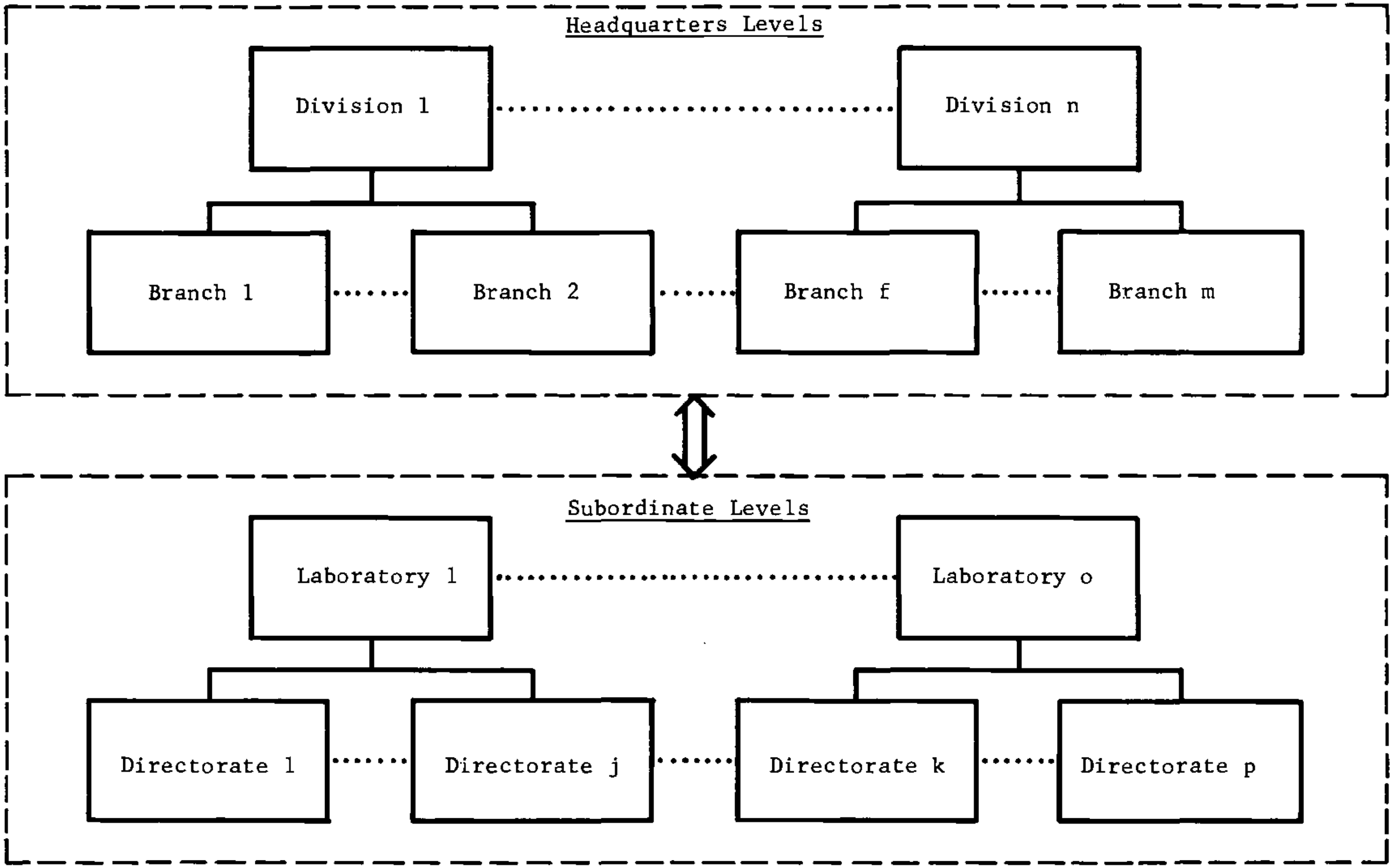

Figure 1. Administrative Hierarchy of a Representative R & D Organization

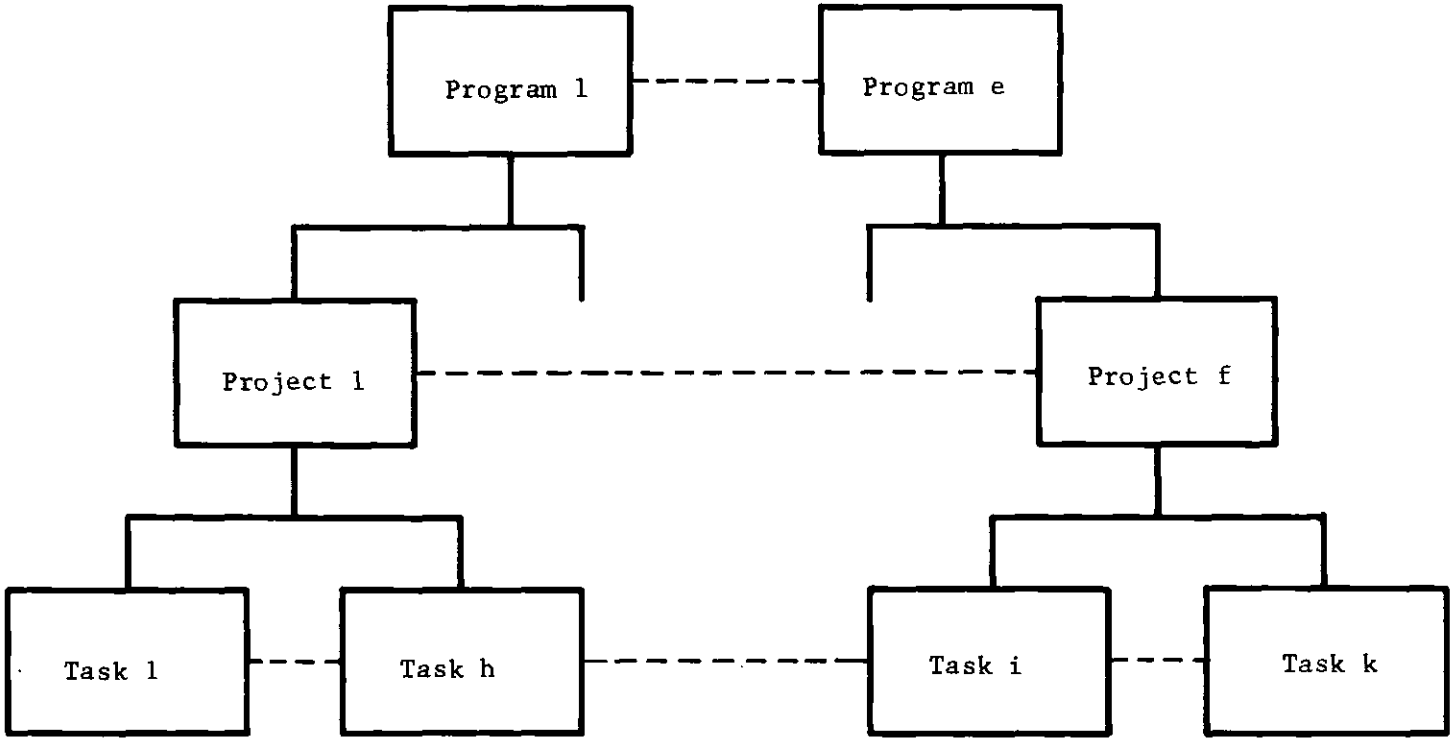

Figure 2. Research Activity Hierarchy of a Representative R & D Organization

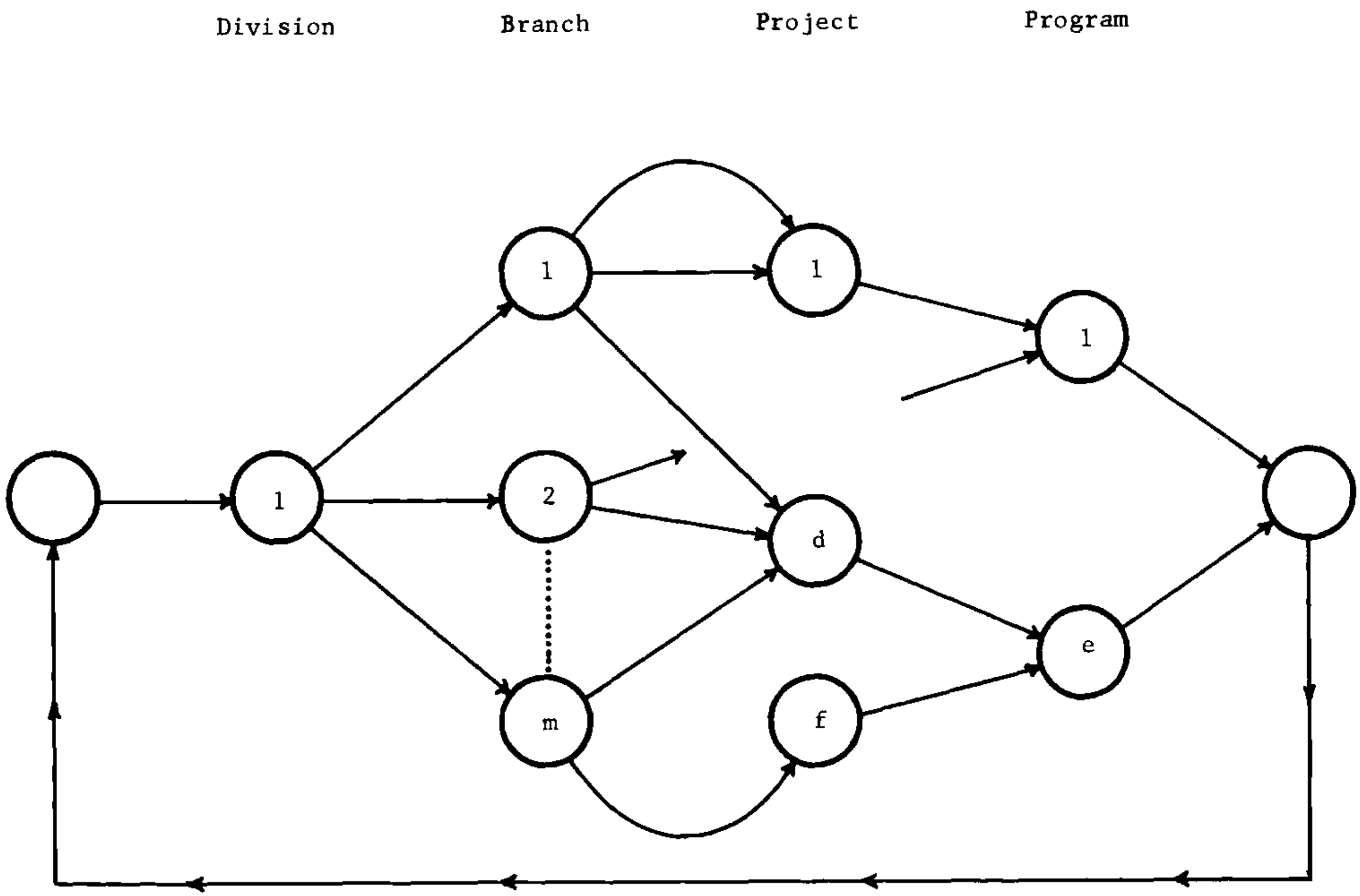

Figure 3. General Network Representation of a Representative R & D Organization

problem with a single commodity (budget dollars) flowing through the network. The necessary characteristic is that subordinate entities can be represented as associated with one and only one superordinate entity. At headquarters, each branch is associated with one and only one division and, at the laboratories, each directorate is associated with one and only one laboratory. The agency's tasks are associated with one and only one project, and projects with one and only one program element. Thus the required characteristic is maintained throughout both hierarchies. The out-of-kilter algorithm is used to solve the rewritten problem [17].

The mathematical formulation of the model to be used is:

$$\text{Min } - v_j(x_j) \tag{1}$$

where $v_j(x_j)$ is the benefit of task j at funding level x_j. The full mathematical formulation of the problem is as follows:

$$\text{Max} \sum_j v_j(x_j) \quad \text{or} \quad \text{Min} - \sum_j v_j(x_j) \tag{2.1}$$

subject to

$$B^- \leq \sum_j x_j \leq B^+ \tag{2.2}$$

$$B_\ell^- \leq \sum_{j \epsilon \ell} x_j \leq B_\ell^+ \quad \forall\ \ell \tag{2.3}$$

$$B_d^- \leq \sum_{j \epsilon d} x_j \leq B_d^+ \quad \forall\ d \tag{2.4}$$

$$B_b^- \leq \sum_{j \epsilon b} x_j \leq B_b^+ \quad \forall\ b \tag{2.5}$$

$$B_j^- \leq x_j \leq B_j^+ \qquad \forall \ j \tag{2.6}$$

$$B_p^- \leq \sum_{j \epsilon p} x_j \leq B_p^+ \qquad \forall \ p \tag{2.7}$$

$$B_e^- \leq \sum_{j \epsilon e} \leq B_e^+ \qquad \forall \ e \tag{2.8}$$

where x_j is the level at which task j is funded

$j \epsilon \ell$; ℓ=1,2,...O identifies the tasks belonging in laboratory ℓ

$j \epsilon d$; d=1,2,...N identifies the tasks belonging in division d

$j \epsilon b$; b=1,2,...M identifies the tasks belonging in branch b

$j \epsilon p$; p=1,2,...P identifies the tasks belonging in project p

$j \epsilon e$; e=1,2,...E identifies the tasks belonging in program e

B^- and B^+ are the lower and upper budgets, respectively

$v_j(x_j)$ is the value of project j funded at level x_j

Benefit Determination

In the model as originally designed and used by Baker et al. [4], the benefit of each task is determined at three funding levels, a minimum funding level, a recommended level, and a maximum level. These are defined as follows: (1) minimum level as that level of funding below which a task cannot be funded without encountering unacceptable consequences; (2) recommended level is that level preferred by the organizational unit consistent with overall guidance; and (3) maximum level is that level above which a task cannot be funded without encountering significantly diminishing returns. For each funding level estimate, a corresponding benefit measurement is supplied.

The following discussion, taken from Baker, et al. [5] is included so that the reader will have a better understanding of the model.

For each j, funding levels x_j^k are specified. There levels of funding (k) have been employed; k=1 is the minimum level (a level below which a unit cannot be funded without encountering unacceptable consequences), k=2 is the recommended level (a level preferred by the organizational unit consistent with overall superordinate guidance), k=3 is the maximum level (a level beyond which a unit cannot be funded without encountering significantly diminishing returns). For each x_j^k, a benefit estimate, v_j^k is specified.

A piecewise linear benefit function, $v_j(x_j)$, is defined through v_j^1, v_j^2, v_j^3, for each j as follows. The analyst specifies an "acceptable percent of error" which is used to test for the linearity of each $v_j(x_j)$. The best least squares regression approximation of $v_j(x_j)$, $\hat{v}_j(x_j)$, is calculated. If the associated least squares estimates of v_j^1, v_j^2, and v_j^3, that is, $\hat{v}_j^1$, $\hat{v}_j^2$, and $\hat{v}_j^3$, are all within the "acceptable percent of error" then $\hat{v}_j(x_j)$ is assumed to be linear. If any one of the $\hat{v}_j^k$ are not within the "acceptable percent of error", then $v_j(x_j)$ is tested to determine if it is concave or convex. Since there are only two linear segments and since all the $v_j(x_j)$ are nondecreasing in x_j, the test consists only of a comparison of the two slopes. Specifically,

$$s_{j1} = \frac{v_j^2 - v_j^1}{x_j^2 - x_j^1} \text{ is compared to } s_{j2} = \frac{v_j^3 - v_j^2}{x_j^3 - x_j^2} .$$

If $s_{j1} > s_{j2}$, then $v_j(x_j)$ is concave. If $s_{j1} < s_{j2}$, then $v_j(x_j)$ is convex. The algorithm works directly with $v_j(x_j)$'s which are linear or concave.

If $v_j(x_j)$ is convex, then the algorithm works with one of four linear approximations. The four linear approximations available are the linear regression approximation and the lines defined by (v_j^1, v_j^2), by (v_j^1, v_j^3), and by (v_j^2, v_j^3). The optimal funding pattern where all convex $v_j(x_j)$'s are estimated by the linear regression approximation is always computed. If so specified in the input data, the computer program will also determine the optimal funding pattern with all convex $v_j(x_j)$ estimated by the (v_j^1, v_j^2) approximation, by the (v_j^1, v_j^3) approximation, and/or by the (v_j^2, v_j^3) approximation.

The original functions, the $v_j(x_j)$, are always used in the calculation of total benefit. The program will not use one of the linear approximations to calculate total benefit or to indicate the benefit of any specific allocation. If more than one linear approximation is used, only the one which yields the largest total benefit is maintained and reported. Furthermore, if alternative optimal solutions exist, the algorithm will select that alternative with the minimum total expenditure.

There is one assumption inherent in this development of piecewise-linear benefit functions. It is that the benefit for an intermediate funding point is given by the piecewise-linear curve at that point.

In the original formulation, this benefit measurement was specified by the use of scoring or comparative methods. However, this method of assigning value to the tasks is not acceptable for this study. For one thing, there would be no explicit way to manipulate the individual's risk tolerance. For another thing, as designed, the model is not a selection model, a capability which is desired for this study, and most importantly, this study is not being conducted within an organization.

As a result of the desire to add a selection capability to the model, it was decided to change the three funding levels used as input data. Instead of using a minimum, recommended, and maximum level as input, the model will now use a zero (not selected), minimum, and maximum funding level. The minimum and maximum funding levels are defined in the same manner as above. The zero funding level will always have an associated benefit of 0. This level indicates no funding or that the task has not been selected. In this manner, the model can be modified to be a task selection as well as resource allocation model.

As stated, the model as originally proposed and used utilized comparative and scoring methods for calculating benefit. This will be changed during this study. Instead, an expected utility or expectation-variance utility measure of benefit will be used. The literature indicates that a quadratic is a reasonably descriptive form for a utility function. The expected utility formulation, $\mu - k\sigma^a$, results in a quadratic form of the utility function. a=2 has been suggested and used by many authors such as Cramer and Smith [10], Farrar [13], Markowitz [26], and Watters [39]. This formulation also appears reasonable from an empirical basis [10]. Furthermore, k is a convenient measure for experimental risk tolerance. Hence, this formulation was chosen for use in this study.

The particular expectation-variance utility function to be used in this study is

$$E(U_j) = \mu_j - k\sigma_j^2 \tag{3}$$

where μ_j is the expected value (mean) of the estimated distribution of returns of task j

σ_j^2 is the variance of the distribution of returns of task j

and k is the risk tolerance factor of the decision maker.

The risk tolerance factor, k, indicates whether the decision maker is risk averse or risk seeking. It further specifies to what degree the decision maker is risk averse or risk seeking. It does this by combining the expectation of the return of a task with some weighting of the variance of this return. A risk intolerant decision maker would have a positive ($k > 0$) risk tolerance factor. He would shade his benefit of a project on the low side of its expected benefit. The more positive k, the more the decision maker will shade his benefit judgment, and hence, the more risk intolerant (conservative) the decision maker is. For the risk tolerant decision maker, the risk tolerance factor, k, will be negative ($k < 0$). This will cause him to shade his benefit so that it will be greater than the expected benefit measurement. The more negative the risk tolerance factor, the higher will be his estimate of the benefit of a given task, hence the more risk tolerant the decision maker is.

Once the model is provided with the expectation-variance utility function and the three funding levels, it then constructs a piecewise-linear benefit function for each task using the three specified funding levels and associated benefit estimates. These piecewise-linear benefit functions are used in determining the optimal funding pattern based upon the slopes of the linear pieces of this benefit function. Due to the fact that the model looks at the slopes of the linear pieces of the benefit function, there is a tendency for the model to function at one of the three specified points; zero, minimum, or maximum funding level. This is due to the change in slope of the value function which is likely to occur at these points.

The model will directly handle any benefit functions which are linear or concave. However, if the benefit function is convex, then one or more linear approximations of this function are used in the algorithm. As a result, the algorithm does not necessarily provide optimal solutions. That is, the results obtained by the use of this algorithm need not be optimal; however, they may be. Because of this sub-optimality, it is not possible to say that the output of the algorithm represents the best possible selection of tasks and allocation of money thereto. Rather, the output should be used as guidance by the decision maker(s) in the decision processes.

Figures 4 and 5 illustrate the resulting network for both models.

> The costs, c_j, associated with the network formulation are the negative of the marginal benefit per dollar expenditure. Therefore, the c_j depend upon the form of the associated $v_j(x_j)$. If $v_j(x_j)$ is linear, c_j is the slope of $v_j(x_j)$ multiplied by minus one. If $v_j(x_j)$ is convex, c_j is the slope of the linear approximation multiplied by minus one. If $v_j(x_j)$ is concave, two arcs are necessary. One arc has constraints of (B_j^-, x_j^2) and a cost, c_j^1, equal to the slope of the line connecting (x_j^1, x_j^2) multiplied by a minus one. The other arc has constraints of (x_j^2, B_j^+) and a cost, c_j^2, equal to the slope of the line connecting (x_j^2, x_j^3) multiplied by a minus one. The c_j's are attached only to the flow of dollars between nodes n and u. An ϵ cost is assigned to the overall budget arc to assure that, if alternative optimal solutions exist which yield the same maximal value, the least cost alternative is selected. That is, it will prevent the flow of additional dollars through the budget arc if that flow does not increase the total value of the research program. All other flows have zero cost (or benefit).

The above discussion is taken directly from "A Budget Allocation Model for Large Hierarchical R & D Organizations," by Baker, Souder, Shumway, Maher, and Rubenstein [5].

Implicit within the development of the model are several assumptions which should be made explicit. This is done in order to make the model more understandable. These implicit assumptions are:

1. A task is funded wholly within a given branch and is associated with only one project.

2. Benefit functions are piecewise linear.

3. The solution of this model does not necessarily provide optimal solutions.

4. This constrained optimization model is a reasonable representation of an individual decision maker.

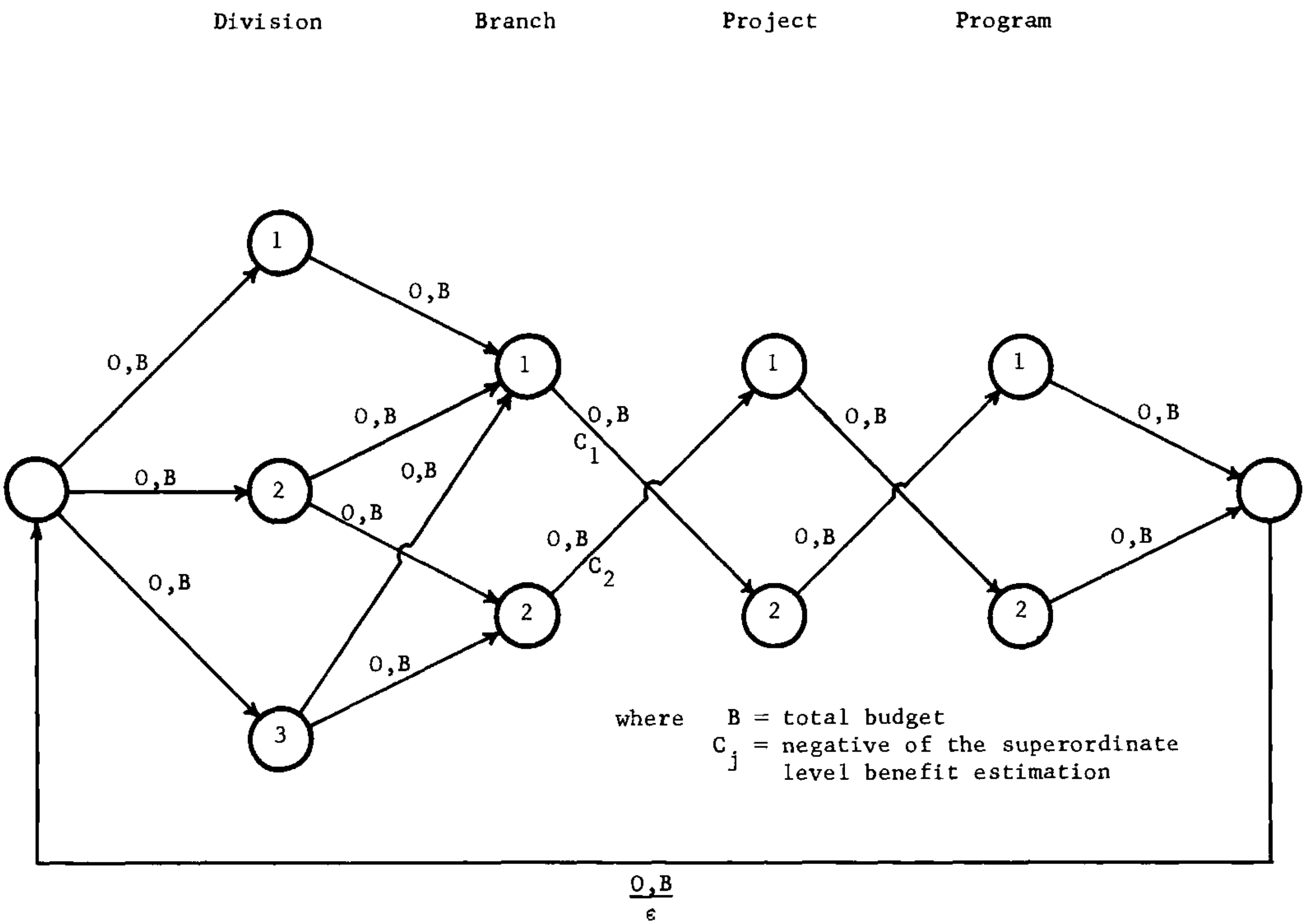

Figure 4. Illustrative Network of Superordinate Level Model

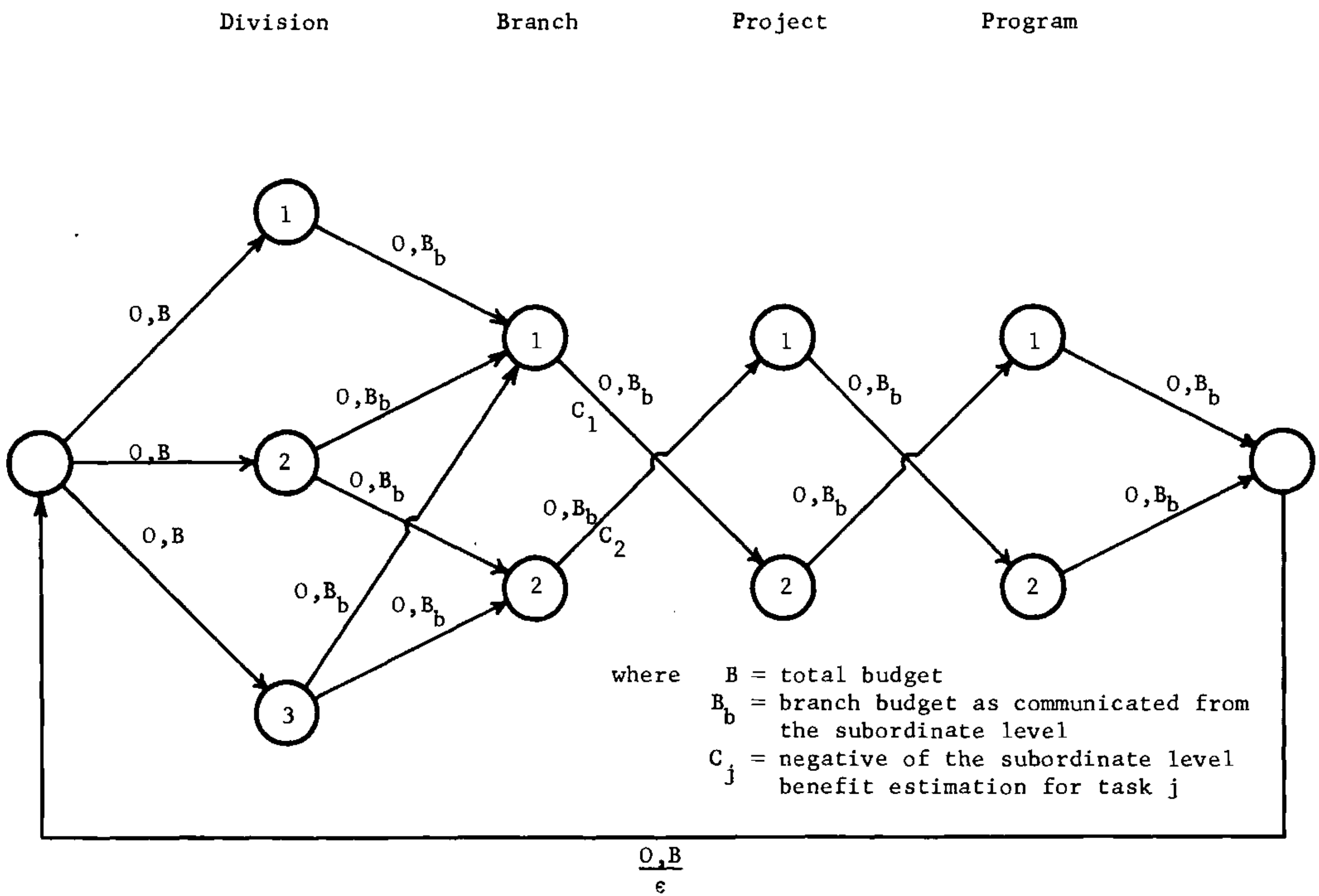

Figure 5. Illustrative Network of Subordinate Level Model

Model Validation

This model, in its original form, was designed to be a predictive and descriptive model of budget allocations in a larger decentralized research organization. It is written as an interactive computer code to be used as an aid by the decision makers in the organization under study by the original researchers. In this original research by Baker, Shumway, Souder, and Maher [4], this model was run in parallel with the budget allocation process as it currently is done. The decision makers did not know of the results given by the model. The results of the model were then compared to the actual budget allocations as made by the decision makers in the organization. Pearson product moment and Spearman rank correlations were used. The product moment correlation measures the relationship between the levels of funding, that is, it considers the difference in levels of funding, alternative by alternative, in the respective budget allocations. In the Spearman rank correlation, the alternatives were rank ordered according to level of funding and these ranks were then correlated. Table 1, presented here by permission of the authors, shows the results of these statistical comparisons. These correlations are determined for two distinct situations. The first set of correlations corresponds to the use of recommended budgets by the branches. The second set of correlations corresponds to a decremented budget.

As can be seen in Table 1, the model as designed is a good predictor of the actual budget allocations. Because of this high correlation of the results of the model to the results of the process being modeled, it seems reasonable to assume that this model is a representative of the process under consideration. The use of such a model in this

Table 1. Correlations of Model Output with Actual Budget Recommendations

Form of Benefit Function	Branch: No. of Alternatives	Correlations at Base Funding: Pearson Product Moment	Correlations at Base Funding: Spearman Rank	Correlations at Decrement: Pearson Product Moment	Correlations at Decrement: Spearman Rank
Simple Comparative	A:12 projects	.7016**	.7228*	.8785*	.8952*
	A2:12 projects	.9355*	.9158*	.9419*	.9383*
	B:5 projects	.8839**	.8000	.9750**	.9747**
	B:27 tasks	.6055*	.5989*	.7020*	.6142*
	C:7 projects	.9624*	.9370*	.9911*	.9370*
	C:25 tasks	.9903*	.9798*	(2)	(2)
	D:11 projects	.5791**	.8909*	.8068**	.9000*
	D:27 tasks	.7067*	.7293*	.8252*	.8342*
Scoring Model	A:12 projects	.7155**	.7579**	(2)	(2)
	A2:9 projects	.9639*	1.000 *	.9891*	.9828*
	B:5 projects	.6549	.7000	.2800	.3591

1. Branch Chief A requested that his benefit scores be divided by cost thus yielding a benefit/cost ratio which was then maximized. The correlations associated with A[1] are based on this adjustment.

2. The data are not available for these correlations.

*Significant at .001.

**Significant at .05.

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current research then makes this research that much stronger because of the apparent validity of the model used [26].

Summary

An abbreviated presentation of the budget allocation model developed and used by Baker, Souder, Shumway, Maher, and Rubenstein [5], and Baker, *et al.* [4] is given here. Also included are modifications of the basic model which are needed for this study. Verification data obtained by Baker, *et al.* [5] in their continuing research with this model are also included. These data show that the model is an accurate predictive model.

CHAPTER IV

DESIGN OF THE EXPERIMENT

Research Questions

The primary purpose of this research is to investigate the effects of individuals' risk tolerance on the selection of and budget allocation to an R & D portfolio in a hierarchical organization. The emphasis will be primarily on multi-level decision processes within this hierarchical organization. Within this general framework, four specific conjectures will be looked at in detail. These conjectures are:

1. The number of tasks funded by a decision maker is a function of that decision maker's risk tolerance.

2. The expected value (mean) of the selected portfolio will be symmetrical about $k=0$.

3. The difference between the superordinate level's portfolio and the subordinate level's portfolio, as measured by "sum of absolute differences" and "portfolio means," is symmetric around $|k_1-k_2| = 0$.

4. The imposition of additional budgetary constraints upon the subordinate will bring his portfolio closer to the superordinate level's portfolio.

The remainder of this chapter will be devoted to explaining how these conjectures will be studied.

Use of the Model

As the model was originally designed and used, the number of levels and the number of entities within each level are variable, up to certain maximum values. These variables are specified as input data to the model. For this study, a two-level administrative hierarchy was used, divisions and branches. Since the object is to study multi-level decisions, the two-level decision process seemed the most logical one with which to begin as it is the simplest of the multi-level decisions. It was arbitrarily decided that, within these two administrative levels, there would be one division and three branches. The activity hierarchy used includes three levels. They are programs, projects, and tasks, respectively. Again, by arbitrary decision, it was decided that there would be one program, nine projects, and 200 tasks. This arrangement allowed for flexibility in the model and also allowed for enough of a base from which meaningful conclusions and insights could be drawn.

The model, in its original design and use, fixed the decision at a given level of the hierarchy. In order to add a multi-level dimension, a sequential solution procedure was used in which outputs from one solution were used as inputs to a new solution. This sequential procedure is presented below step by step and is flow-charted in Figure 6.

1. Fixing the benefit function at the superordinate level and putting no constraints on any budgets, division, branch, or task, solve the model. From this output one obtains a portfolio (selected by the superordinate level) and budgets to all entities within the organization.

2. Take the budgets obtained in the superordinate level solution of the algorithm (above) and use them as upper limits of the budgets to

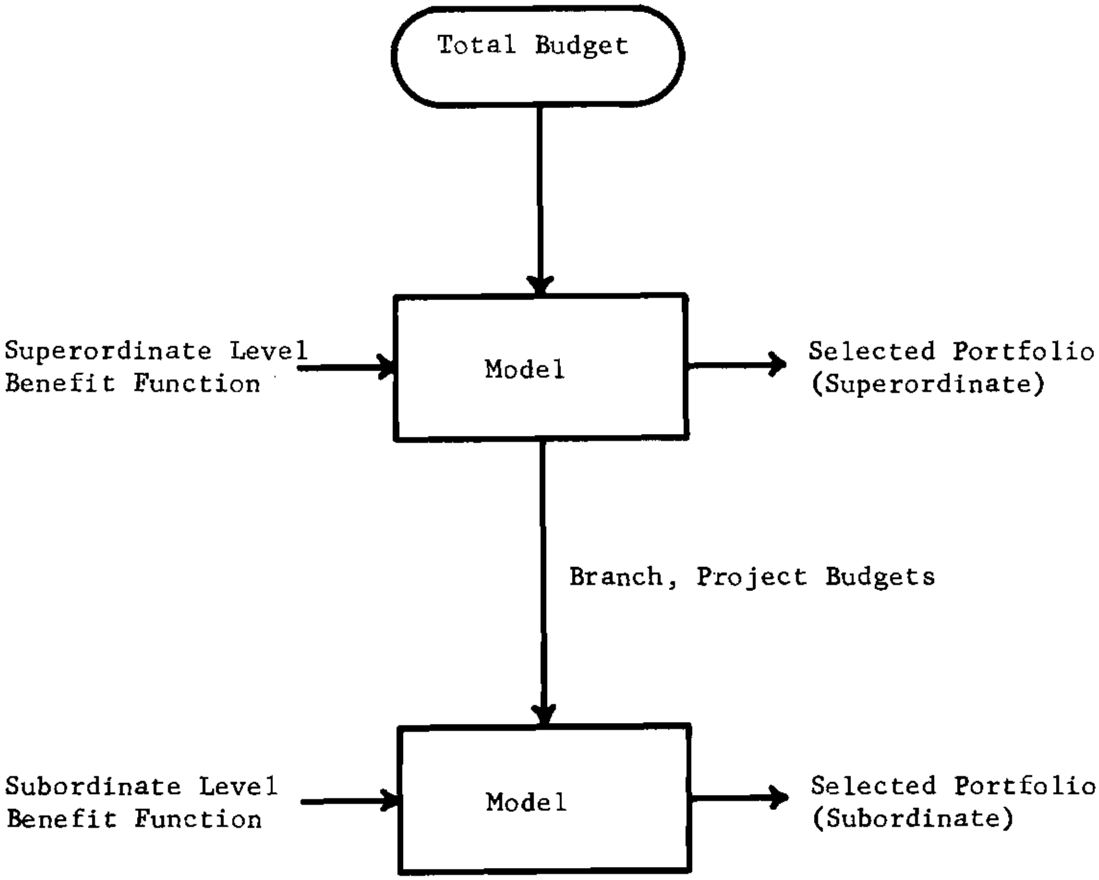

Figure 6. Flow Diagram Depicting the Sequential Solutions of the Model

corresponding organizational entities in the subordinate level solution. Also, change the benefit function to that of the subordinate level. Resolve the model. As output, we again get a portfolio. This portfolio is a result of the multi-level decision process and will be compared to that portfolio selected by the superordinate level. It is from these comparisons that conclusions concerning the multi-level decision process will be inferred.

In step 2, the resolution of the model using the benefit function of the subordinate level, the budgets which can be passed from the superordinate level solution are branch, project, and task. In this study, both branch and project budgets will be passed from one solution to the other, but task budgets will not. Were this done, the subordinate level solution, by the nature of the budgets or constraints, would be limited to only one feasible solution: the same solution obtained in step 1. Thus, no use would be served by passing task constraints from the superordinate level solution to the multi-level solution.

Method and Indices of Portfolio Comparison

The objective of this research is to determine the effects of a decision maker's risk tolerance on the portfolio he selects. Moreover, it is desired to look at these effects as they apply to decisions made in a hierarchical organization. The main thrust of the research is then on the comparison of portfolios and the determination of the "amount" by which they differ. This is particularly crucial in the case of the multi-person decisions and specifically in the testing of conjectures 3 and 4. In these cases, it is desired to compare the portfolio selected by the superordinate to that selected by the subordinate who has been influenced by the superordinate. Recalling that the model is solved sequentially, first for the superordinate and then for the subordinate, we then have the two portfolios needed for comparison readily available.

No single index or number was found which characterized the differences in portfolios when they were compared. Therefore, a number of indices will be used. These indices when considered together do give a complete picture of the differences in the portfolios. One of the indices to be used is the number of tasks selected (funded) in each portfolio. This index will point up situations in which one decision maker funds a particular task and the other does not. Moreover, this index is needed to obtain results for the testing of the first conjecture. The second index to be used will be the mean and variance of the selected portfolios. Since all tasks considered in this study are independent by assumption, the mean of the portfolio will merely be the sum of the means of the tasks at the funding levels at which they are selected. The same will be true for the variances. The use of this index will show differences in the riskiness of the selected portfolios as well as differences in their expected values. The final index to be used will be the sum of absolute differences. This index will be indicated by the abbreviation SAD. This index will compare the two portfolios task by task. If the superordinate and subordinate differ on their funding allocations to a particular task, the absolute value of that difference will be noted. This includes tasks selected by one and not the other. This comparison will be made for each of the tasks in both portfolios. These absolute differences will then be summed and this sum (SAD) will then be used as an index. The value of this index is that it allows a single number to be attached to a task by task comparison of the portfolios. This will show differences in the portfolios do exist even if both fund the same tasks exactly.

Data

The model requires benefit judgments and budgets as input data. In order to operate, the model requires one other piece of information. This other required information is task data. The model requires a pool of tasks and associated benefit judgments, or in the case of the modified model, estimates of the mean and variance of each task at each of three funding levels; zero, minimum, and maximum. The minimum and maximum funding levels must also be specified. The reason that the modified model only requires estimates of the mean and variance of task returns at each funding level is that, in the modified model, the calculation of benefit is accomplished by the model from these data.

In this study, there are two sets of data which are uninteresting as the results which would be obtained from the use of these sets of data are predictable. One of these sets of data is data in which all the task means are so large that they always overshadow the task variances. A decision maker operating on data of this type would consistently choose the tasks with the largest means, no matter what his risk tolerance is. The other type of data is data in which the variances are so large that they overpower the means of of the tasks. With data of this type, any decision maker will choose the high mean-high variance tasks. The expectation-variance formulation of expected utility is the reason for the above behavior. Recall that, in this formulation, the expected utility is equal to the mean minus some fractional multiple of the variance. Thus, in tasks where the mean dominates the variance, the tasks will be selected on the basis of their means. In the cases where the variance dominates, the tasks selected will be those with the largest means and variances.

For this study, it was desired that the data used not be wholly of either type described above. The desired data would have the property that sometimes the mean would dominate, sometimes the variance, and sometimes they would just trade off. It is this type of data which is used in the research.

Generation of the Data

The data used in the research were randomly generated. A base of 200 tasks was generated and then the decision level operated upon this 200 task base. Different parameters of the decision maker or his feasible solution set were varied in the different experiments.

The method used for generating the data was to set an interval on which the values of a particular parameter of the data could range. This interval was then looked upon as a uniform distribution of the possible values of that parameter. Realizations of this distribution were then selected randomly by the use of a 0-1 random number generator [22]. The data parameters specified in this maneuver were the minimum funding level, the mean at the minimum funding level, and the variance at the minimum funding level for each of the 200 tasks.

To obtain the values for the maximum funding level, the mean at maximum funding and the variance at maximum funding, a slightly different method was used. An incremental range was specified for each of these parameters. This range was then considered to be a uniform distribution of the possible values for these parameters. A realization of these increments was then randomly made by use of the 0-1 random number generator. The increment chosen for a particular parameter was then added to its corresponding value at the minimum funding level.

A brief example may help to clarify this procedure. Let us suppose that the range of minimum funding levels is uniform on the interval 100-200. Let us further suppose that a random realization of this distribution is 179. In other words, the minimum funding level of the task under consideration is 179. We now want to find the maximum funding level for this task. Assume that the incremental funding values are uniformly distributed along the interval 50-100. Let us further assume that a random realization of this distribution is 61. The maximum funding level for this task is then the minimum funding level plus the funding increment or 179 + 61 which is 240. This process is followed for each of the parameters which must be specified for each of the 200 tasks in the data base. Table 2 gives the ranges of the values for all parameters of the data used.

Table 2. Ranges of Parameters of the Data Used in the Study

Parameters	Range at Zero Level	Range at Minimum Level	Range at Incremental
Funding	0	400-900	100-400
Mean	0	1000	300-700
Variance	0	300-700	$^{-}$300-300

Two other parameters also had to be specified for each task. These were to which project the task belonged, and to which branch the task belonged. As there are nine projects, the tasks were randomly assigned to one of the nine projects so that they were uniformly distributed among the projects. This resulted in each project containing approximately the same number of tasks since the tasks were distributed among projects by a uniform distributor. The same procedure was followed for assigning the tasks among the three branches. One further parameter had to be specified. This was to which branch each project belonged. This was done arbitrarily since the only purpose of this is to allow completion of the network and no effects on the results could be caused by this arbitrary assignment. The assignment was that projects 1, 2, and 3 belonged to branch 1; projects 4, 5, and 6 to branch 2; and projects 7, 8, and 9 to branch 3.

Summary

The research questions to be answered are presented in this chapter. This includes the manner in which the model is to be used, the nature and values of the data used, the measures of portfolio similarity used, and the procedure used to generate the data used in this research.

CHAPTER V

RESULTS

Review of the Model

The model in its original form will solve the selection and allocation problem for one level of an organization. This is accomplished by specifying the objective or benefit estimation model to be used in that solution. For use in this research, the model will be used in a sequential manner in order to achieve the property of a multi-level organization, which is so necessary to this work.

The first step in this sequential process is to solve the model using the superordinate's benefit function. The only other information given is the total budget with which he has to work. From this solution of the model, a portfolio of tasks to be funded and the funding levels for these tasks are obtained. Recall that each task is associated with one and only one branch and with one and only one project.

Next, from this solution of the model, branch and project budgets can be obtained. Recall that these budgets become constraints in the next solution of the model. These budgets are then communicated to the subordinate level for use in a subsequent solution of the model. By summing the funding levels of all tasks funded (by the superordinate) within a branch, a budget for that branch is obtained:

$$B_b = \sum_{t \epsilon b} \hat{x}_t \quad \forall\ b$$

where B_b = branch budget for branch b

$\hat{x}_t$ = funding of task t when superordinate's benefit functions are utilized

$\bar{b}$ = set of all tasks in branch b.

This branch budget is then the budget which the superordinate level communicates to that branch. This is done for all branches so that each branch has its own budget. This same sort of calculation is carried out for each project in the activity hierarchy. By summing over all tasks belonging to a particular project, a project budget can be calculated:

$$B_p = \sum_{t \epsilon \bar{p}} \hat{x}_t \quad \forall \; p$$

where B_p = budget of project p

$\hat{x}_t$ = funding of task t when superordinate's benefit functions are used

$\bar{p}$ = set of all tasks in project p.

Once these budgets have been obtained, the next step in the sequence can commence.

It now must be decided which of these budgets, or constraints, are to be communicated to the subordinate level. In some cases only branch budgets are communicated and in others both branch and project constraints are communicated from the superordinate level to the subordinate level. These budgets are communicated as constraints by inputting them as upper bounds to the respective branch or project in the second solution of the model.

The second solution of the model is the final step of this sequence.

The objective function used is that of the subordinate level. The appropriate constraints from the superordinate level or first solution of the model are also applied for this solution. It is the application of these constraints which adds the multi-level capability to the model. The portfolio obtained from this second solution can then be compared to the portfolio obtained from the first solution and comparisons can be made. It is through these comparisons that insights can be gained.

The Effects of Constraints

When constraints are imposed on the model, as was discussed in the previous section, the problem goes from a single decision problem to some number of independent decision problems which must be solved. This number is equal to the number of branch and/or project budgets which are imposed upon the model. Two basic outcomes can result when these budgets are communicated. One outcome is that the constraint, the budget from the superordinate level, is greater than the amount which would have been allocated to that branch or project if the model were solved with superordinate benefit functions and no constraints on the branch (or project). The second event is that the constraint is less than that which would have been allocated by an unconstrained model with subordinate benefit functions. Each of these outcomes can cause two different behavior patterns in the model.

If the imposed budget is greater than would have been allocated by the unconstrained subordinate model, the additional funds made available by the imposed budget will be used since returns are non-decreasing. The funds can be used to fund additional tasks which would not be funded within that branch or project had the extra funding not been available and/or to increase the funding of the tasks already funded. These two behavior patterns are not mutually exclusive and some combination of both behaviors can be used to exhaust the budget. The objective function of the decision level will be the determining factor as to how much of the additional funds are spend in each manner. The basis for determining how these additional funds will be allocated is incremental return. The precise method of how this works will be developed in the next section.

Two types of behavior also are possible when the communicated budget is smaller than would have been determined by the unconstrained subordinate model. These behaviors are the converse of the behaviors seen in the previous paragraph. When faced with the problem of allocating less funds within a branch or project than it would with an unreduced budget, the model, as one alternative, can fund fewer tasks, e.g., not fund some tasks that it normally would fund if it had an unconstrained branch or project budget. The other alternative is to fund the same tasks as it would have had the budget not been decremented, but to fund these tasks at reduced levels. Again, the model can employ both alternatives within a given project or branch. The determining factor is incremental return. The model will choose that alternative or combination of alternatives which will reduce its benefit estimate the least, just as in the previous case it would choose the course of action or combination of courses of action which would increase its benefit estimate the most.

The Algorithm

As was explained, the algorithm used to solve this network model is the out-of-kilter algorithm. This algorithm uses an iterative solution procedure. It is this iterative procedure which can be used to explain the behavior of the model under conditions of increased and decreased budgets.

The model selects tasks for funding and sets the level of funding based upon the benefit estimation balanced against the expenditure. In an unconstrained environment, the model selects first that task and the level of funding for that task which yields the best overall benefit per dollar allocated. The model then scans all remaining tasks and continues to select tasks and fund them in a decreasing order of benefit per dollar allocated. This process continues until either all tasks have been selected and funded or the budget has been exhausted. Given the situation described in the preceding section in which the budget communicated from the superordinate level to the subordinate level is greater than the unconstrained subordinate allocation, it is the iterative nature of the solution procedure which explains how these extra funds are allocated. After selecting and funding those tasks which it would if it were not constrained, the model then looks at the incremental benefit to be gained from funding each of the remaining tasks. It also looks at the incremental benefit to be gained from increasing the funding of any task which has been previously funded. This incremental benefit estimation is based on the objective or benefit-estimation function used in the solution procedure. The model then selects that task to fund (or increase the funding level of some previously funded task) which has the best incremental benefit. The model continues in this manner until all available funds have been committed to tasks.

The actions of the model when operating under reduced budgets are also explicable by its iterative nature. When a subordinate level seeks to determine its portfolio and has had a branch or project budget communicated to it from the superordinate level which is smaller than it would itself have allocated to that branch or project, the model reacts in one or both of the ways described in the previous section. The model, based upon the benefit-estimating function it is using, develops a priority list for the funding of tasks, and at what levels to fund them. That is to say, it first funds that task at the level which yields the greatest benefit. The model continues to fund tasks at the level which yields the greatest benefit per dollar in a decreasing order of benefit/dollar until all funds within a given branch or project are allocated. In this case of a decremented budget, the model will fund tasks in the same manner as before, but it has less money to allocate. The result is that fewer tasks are funded and additional tasks are funded at reduced levels from what the subordinate level might otherwise have desired.

The Conjectures

The results of this study can be broken down into two classifications: (1) those results dealing with single level decision processes; and (2) those results dealing with multi-level decision processes. These results will be discussed in the remainder of this chapter. First, however, a note about the use, or lack thereof, of statistics in this work.

The nature of this work is exploratory. Its goals were to gain insights into an extremely complex process. It was felt that, due to the nature of the data used, many of these insights and trends may not have been statistically significant. Thus, had statistical tests been the criteria, many interesting and important results may have been missed. In any subsequent study of some of the insights gained from this work, statistical tests may be mandatory in order to fully explore and understand the insight, but in this research, they may have tended to mask more than they helped to uncover.

The first class of results to be discussed will be those dealing with single level decision processes. There are two conjectures of this type which were examined. They are:

1. The number of tasks funded by a decision level is a function of that decision level's risk tolerance.

2. The expected value (mean) of the selected portfolio will be symmetric about k=0.

This first conjecture simply says that the risk tolerance of a decision level in some way affects the total number of tasks funded by that decision level. The results of this study support this conjecture. These results show that the more risk tolerant the decision level, the greater the number of tasks it funded. The results are shown in Table 3 and Figure 7. For this to happen means that the risk intolerant decision level funded tasks at higher levels than did the risk tolerant decision level.

Table 3. Number of Tasks Funded in Portfolio by a Single Level Decision Process with Risk Tolerance, k

Risk Tolerance (k)	No. Tasks Funded
1.0	120
0.5	129
0.1	130
0	132
-0.1	133
-0.5	139
-1.0	144

Recalling the form of the expectation-variance benefit estimating function, one can see that the manner in which task means and variances change with funding level plays an important role in determining at which level a task will be funded. It is very possible, due to the nature of the data used, that when increasing the funding level of a task, the increase in the task mean will offset any change in task variance. If this is the case, then the risk intolerant decision level will prefer to fund at the higher levels almost all of the time, while the risk intolerant decision level will prefer the reduced funding levels. Earlier in this chapter, the iterative nature of the model was discussed. Another possible reason for the behavior noted here is the nature of the data used. Referring back to Table 2, one can see that, at the maximum funding level, the variance of the task return will decrease from the variance at the minimum funding level approximately 50 percent of the time. The tasks where the variance decreases at maximum funding levels become extremely attractive to the risk tolerant decision maker at this increased funding level. Conversely, these tasks which may be attractive to the

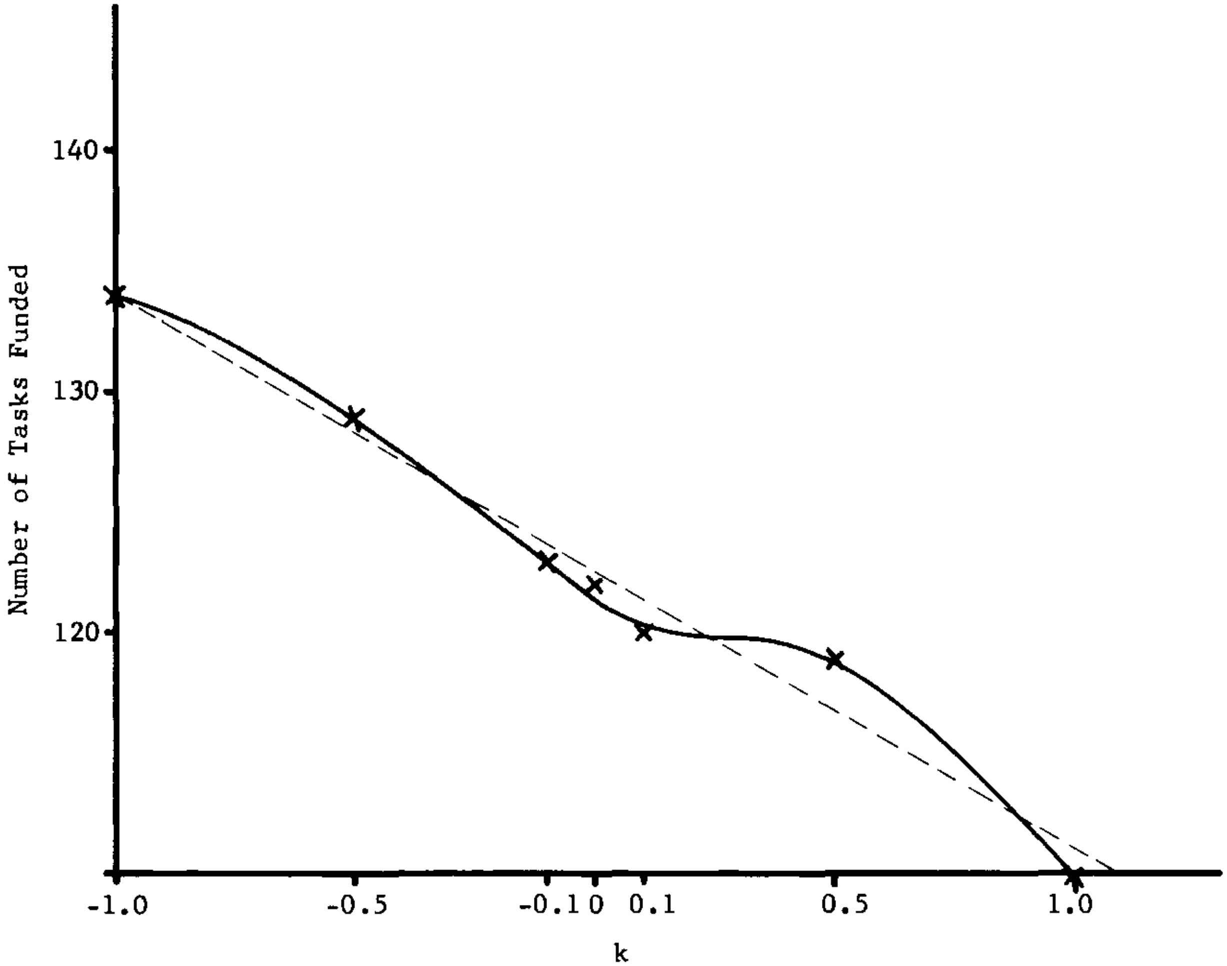

Figure 7. Total Number of Tasks Funded vs. the Inherent Risk Factor (k)

risk tolerant decision maker at the minimum funding level, become less attractive at the maximum funding level. The above property of the data, the benefit function used, and recalling this earlier discussion, it now becomes reasonable to expect the model to give the results that it has.

The second conjecture about single level decision processes deals with the portfolio expected values. The conjecture says that the expected values of the portfolios will be symmetrical about k=0. The results do not fully support this conjecture. The portfolio with the highest expected value had k=-0.1. Moreover, the portfolio with k=0.5 had a higher expected value than did the portfolio with k=-0.5. The same was true for the portfolio with k=-1.0 as it had a greater expected value than the portfolio chosen with k=1.0 (see Table 4 and Figure 8).

Table 4. Portfolio Expected Values for the Differing k Levels

k	Portfolio Expected Value
1.0	182663
0.5	190952
0.1	191724
0	191916
-0.1	192087
-0.5	189959
-1.0	185254

However, these results are explicable. Recall from an earlier chapter that the model is a suboptimization model. It requires certain forms of benefit functions to be approximated in order for the algorithm to be effective. Noting that the results are close to showing the con-

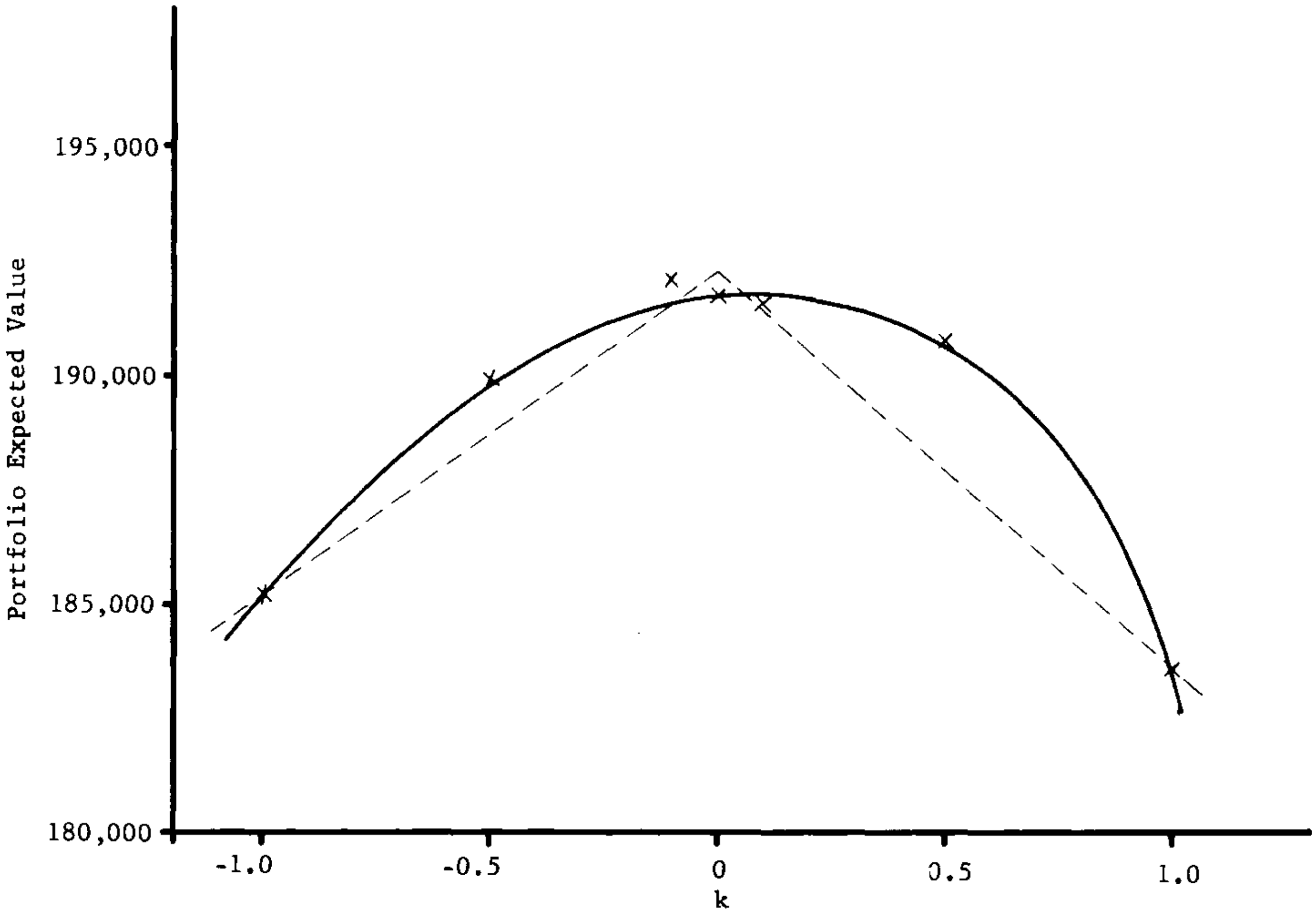

Figure 8. Portfolio Expected Value vs. Inherent Risk Factor (k)

jectured symmetry. The deviations shown can be attributed to these approximations. Another possible cause of these deviations would be the manner in which the data were generated.

However, if these do not explain the deviations from the expected symmetry, then the results indicate that the benefit estimation based on risk tolerance or risk intolerance was more sensitive to portfolio expectations. One could then conclude that whichever risk tolerance situation was favored by the benefit formulation was less dependent on task variance than the other. This would force the definition of a new parameter, other than variance, to indicate task riskiness.

The other results drawn from this study concern multi-level decisions. The two conjectures dealing with multi-level decisions researched are:

3. The difference between the subordinate level's portfolio and the superordinate level's portfolio is symmetric about $|k_1-k_2|=0$, where k_1 is the superordinate level's risk tolerance and k_2 is the subordinate level's risk tolerance.

4. The imposition of additional budgetary constraints upon the subordinate level by the superordinate level will bring his portfolio closer to the superordinate level's portfolio.

Before going into these results in detail, some comments about the sensitivity of the model with respect to differences of values of the risk propensity on different levels are called for. The risk tolerance settings for both superordinate and subordinate levels are shown in Table 5.

Table 5. Sets of Superordinate (k_1) and Subordinate (k_2) Levels of Risk Tolerance to Determine Model Sensitivity

	k_1	k_2
Set 1	1.00	.80 .90 .95 .99
Set 2	.50	.30 .40 .45 .49
Set 3	- .50	- .70 - .60 - .55 - .51
Set 4	-1.00	-1.20 -1.10 -1.05 -1.01

The results of this sensitivity test show that the model is sensitive to differences of .05 or greater for the given data set. The model cannot differentiate between values of less than .05 and the portfolios which result from the single level superordinate process and the multi-level process are identical. Table 6 and Figures 9 and 10 summarize these results.

These results also show that, as the absolute difference in risk tolerance values between superordinate and subordinate decreases, so do the portfolios chosen by the decision processes using the benefit estimations of the respective levels. The sensitivity limit of a .05 absolute difference in risk tolerance is sufficient for the given set of data to allow the investigation of the conjectures concerning the multi-level decision processes. These conjectures will now be discussed.

The first of the multi-level decision process conjectures deals with differences in risk tolerance between levels. It states that the difference between portfolios should be symmetric with respect to the difference between the risk tolerance values of the different levels. This is to say that the difference between the single level superordinate portfolio with k_1 and with multi-level superordinate-subordinate portfolio chosen with k_1 and k_2, respectively, will be the same as the difference between the single level superordinate portfolio chosen with k_2 and the multi-level superordinate-subordinate portfolio chosen with k_2 and k_1, respectively. The values of k_1 and k_2 used to evaluate this conjecture are presented in Table 7.

The results as shown in Table 8 do not support this conjecture

Table 6. Number of Tasks Funded with Subordinate Risk Tolerance Level k_2 at Associated Superordinate Risk Tolerance Level k_1

Risk Tolerance Level	$k_1 = 1.00$	$k_2 = .80$	$k_2 = .90$	$k_2 = .95$	$k_2 = .99$
Number Tasks Funded	120	126	124	123	120
Risk Tolerance Level	$k_1 = .50$	$k_2 = .30$	$k_2 = .40$	$k_2 = .45$	$k_2 = .49$
Number Tasks Funded	129	131	129	131	129
Risk Tolerance Level	$k_1 = -.50$	$k_2 = -.70$	$k_2 = -.60$	$k_2 = -.55$	$k_2 = -.51$
Number Tasks Funded	139	143	142	141	139
Risk Tolerance Level	$k_1 = -1.00$	$k_2 = -1.20$	$k_2 = -1.10$	$k_2 = -1.05$	$k_2 = -1.01$
Number Tasks Funded	144	147	144	144	144

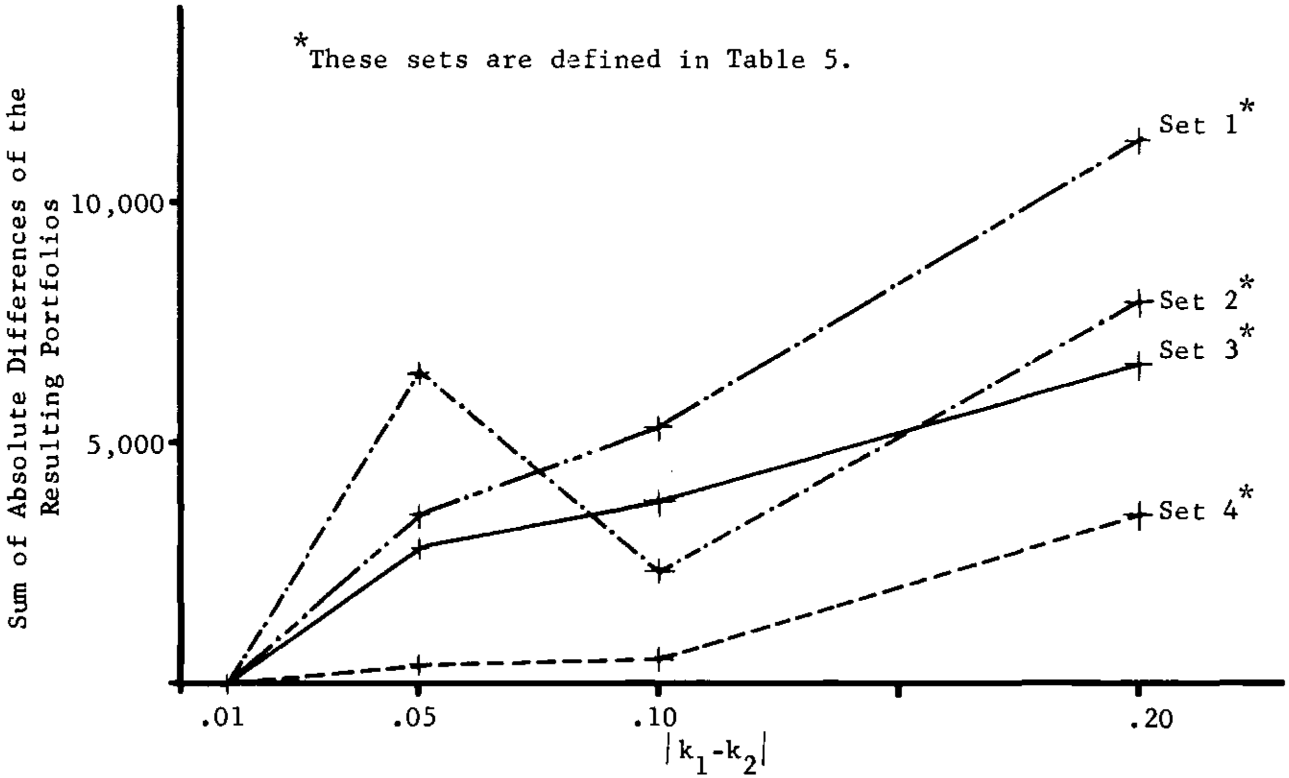

Figure 9. Absolute Difference in k between Superordinate and Subordinate Levels vs. the Sum of Absolute Differences between the Respective Portfolios

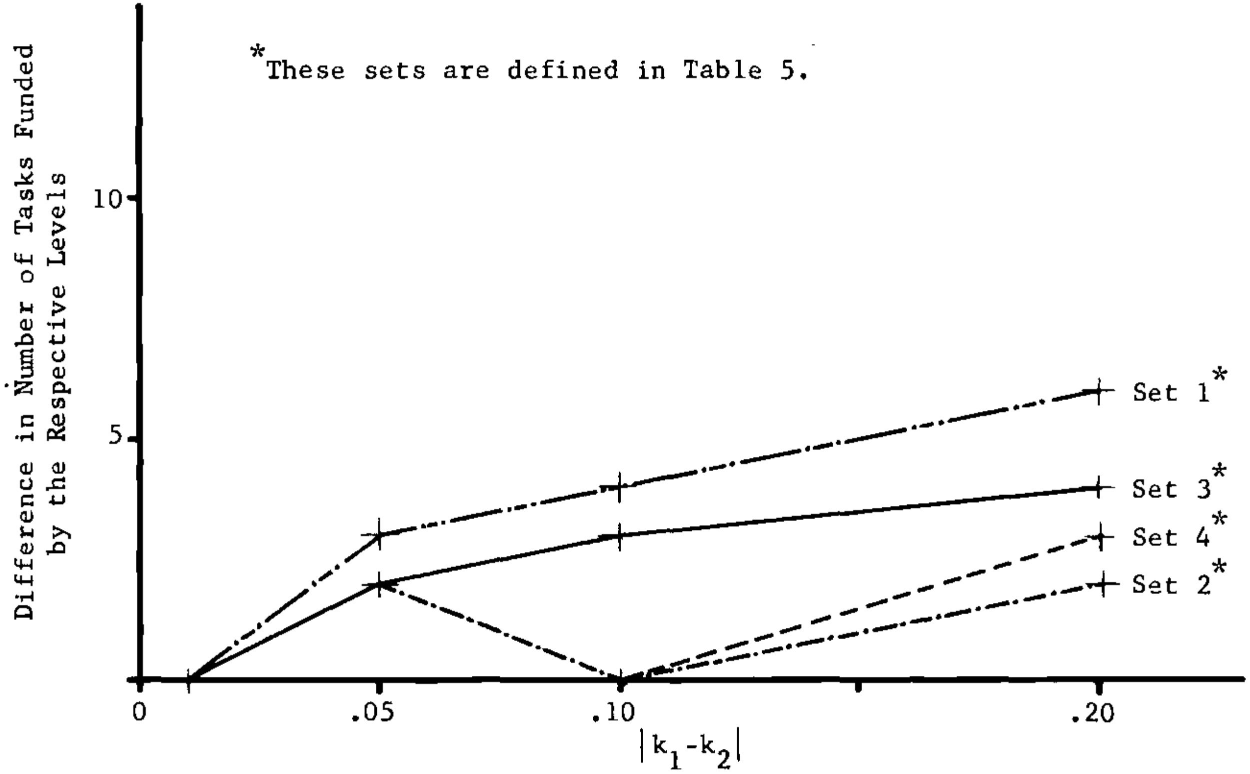

Figure 10. Difference in Number of Tasks Funded by Superordinate and Subordinate Levels vs. Absolute Difference in k between the Respective Levels

Table 7. Risk Tolerance Factor Settings Used to Test Conjecture 3

Run	k_1	k_2
1	1.00	-1.00
2	-1.00	1.00
3	.50	- .50
4	- .50	.50
5	.10	- .10
6	- .10	.10

Table 8. Results from the Test of Conjecture 3 for Varying Superordinate Risk Levels (k_1) and Subordinate Risk Levels (k_2)

	Portfolio Mean	Portfolio Variance	No. Tasks Funded	Sum of Abs. Differences
k_1 = 1.00	182663	48549	121	60430
k_2 = -1.00	183775	79478	146	
k_1 = -1.00	185254	82138	146	63294
k_2 = 1.00	183055	50864	123	
k_1 = .50	190960	58319	129	32200
k_2 = - .50	189615	73631	141	
k_1 = - .50	189952	74705	139	32940
k_2 = .50	189384	58543	129	
k_1 = .10	191724	63513	130	6320
k_2 = - .10	191087	66575	134	
k_1 = - .10	192087	67254	133	6660
k_2 = .10	190413	62426	131	

at all. They show that the portfolios differ by a greater amount when the benefit function of the superordinate level is risk tolerant. Both the sum of absolute differences index and the portfolio means and variances suggest this asymmetry of the portfolio differences.

When the benefit function (objective function) of the superordinate level uses a risk intolerant risk propensity, $k > 0$, the resulting difference between that portfolio and the portfolio selected by the multi-level process using a risk tolerant subordinate is always less than the difference between the portfolio by a risk tolerant superordinate benefit function and the portfolio selected by a multi-level process using a risk intolerant subordinate benefit function. The size of this difference is very small.

The funding characteristics found and discussed in the first conjecture are found when the model represents a single level decision process with a risk averse benefit function. Recall from earlier in the chapter the steps necessary to make the model represent a multi-level decision process, in particular, how the budgets (constraints) are calculated. With the imposition of these budgets upon the subordinate level, it is now forced to find tasks branch by branch.

The communicated branch budgets are either greater than or less than the budget that would be expended by the subordinate level within that branch had it been unconstrained. If this communicated budget is larger, the behaviors which result are then the same as is discussed earlier in this chapter. The risk tolerant subordinate level tends to allocate these additional monies to tasks which it had not funded before.

This occurs because of the incremental benefit and relates back to conjecture 1 and the data used. Conversely, when the communicated budget is smaller, then the model again behaves as discussed earlier in this chapter; that is, that the subordinate level will find fewer tasks than it would if it were unconstrained.

However, when the risk tolerances of the levels are reversed, that is, the superordinate level is risk tolerant and the subordinate level is risk intolerant, the model does not behave in precisely the same manner although the same alternatives are available. In the case of a decremented budget, the model behaves in the same manner as above, fewer tasks are funded and some tasks are funded at reduced levels. Under conditions of incremented budgets, the model chooses increased funding levels rather than fund additional tasks, when the risk tolerances are as stated. It will devote most of the additional funds to increasing the funding levels of tasks previously funded because of the nature of the benefit function and the properties of the data, as discussed in conjecture 1.

What has happened is that, when the superordinate level uses a risk intolerant benefit function and the subordinate level uses a risk tolerant benefit function, the imposed budget caused the subordinate level to fund more tasks than it would have without the budget imposed on it from the superordinate level. This is a "net" figure as in some branches fewer tasks were funded in the constrained situation than in the unconstrained situation. The same situation appears to occur when the superoridinate level uses a risk tolerant benefit function and the subordinate level uses a risk intolerant benefit function. Because of tasks being added, funded at increased levels, deleted, or funded at decreased levels, and some peculiarity of the data used, the differences in the portfolios when the superordinate level uses a risk intolerant benefit function and the subordinate level uses a risk tolerant benefit function are less than the corresponding differences found when the superordinate level uses a risk tolerant benefit function and the subordinate level uses a risk intolerant benefit function. These differences are slight and taken with the other indices used indicate no particular trend. Thus the results do not support the conjecture, but no clear trend was found.

Another insight can be gained from the data presented in Table 8. In every case presented in this table, the variance of the multi-level portfolio is characteristic of the risk tolerance of the subordinate level, i.e., the subordinate level in the multi-level decision process is controlling the riskiness of the final portfolio. In this sense, riskiness will be equaled to a higher variance; so the higher the portfolio variance, the more risky the portfolio. While the variances represent the characteristic of the subordinate level's risk tolerance, the means of the multi-level portfolios have remained consistent with those of the superordinate level portfolios. Since it is the subordinate level which is actually selecting the tasks in the multi-level decision process, it seems only reasonable to expect the selected portfolio to reflect the risk tolerance of this subordinate level.

The last conjecture investigated was the most interesting; the results obtained were the most surprising. This conjecture deals with the imposition of budgetary constraints by the superordinate level on the subordinate level in order to control the actions of the subordinate. The conjecture says that the imposition of these constraints upon the subordinate level will cause the subordinate level to select a portfolio more in line with that desired by the superordinate level than he would do were the subordinate level left unconstrained. This study only looks at the extension of these constraints to the project level.

Budgetary constraints are not extended to the task level because, if the funding of the superordinate level were imposed on every task for the subordinate level, then the subordinate level would have absolutely no option, no matter how different its benefit function might be from that of the superordinate level, other than to exactly duplicate the portfolio and funding pattern of the superordinate level. However, rather than pass from superordinate level to subordinate level, a specific budgetary constraint, a range could be passed from the higher level to the lower level instead. The effects of using a range for the budgetary constraints are not known; however, these effects would be related to the size of the range used. This type of constraint was not used.

The results do not lend support to this hypothesis at the project level. In some instances, the additional constraints did bring the portfolios closer together (see Table 9). In other instances, the added budgetary constraints had no effect on the difference between the portfolios, neither moving them closer together nor farther apart (see Table 10). Finally, in the remaining cases, the additional constraints actually caused the difference between the portfolios to increase (see Table 11). An operational rule was used to decide into which category these port-

Table 9. Results in which Additional Constraints Imposed at the Project Level Caused the Portfolios to Become More Alike

		Superordinate Portfolio			Subordinate Portfolios							
					Branch Constraints				Project Constraints			
k_1	k_2	Mean	Variance	No. Funded	Sum of Abs. Dif.	Mean	Variance	No. Funded	Sum of Abs. Dif.	Mean	Variance	No. Funded
.50	1.00	190952	58620	129	24620	182998	50245	123	19482	181544	50484	127
.10	1.00	191724	63513	130	38844	183758	50952	122	30928	180364	50288	124
.10	.50	191724	63513	130	14988	189574	58407	128	11428	186352	58129	129
.10	- .10	191724	63513	130	6320	191087	66575	134	4678	187170	64363	134
.10	-1.00	191724	63513	130	33246	183849	80726	146	31240	178029	76350	145
- .10	1.00	192087	67254	133	44346	184267	50916	122	38846	180750	51208	128
- .10	.10	192087	67254	133	6660	190413	62426	131	6000	191223	64352	133
- .10	-1.00	192087	67254	133	26074	184131	79959	143	25546	183620	79479	149
- .50	1.00	189952	74705	139	55168	183917	50418	122	48626	180487	49809	128
- .50	- .10	189952	74705	139	15452	191195	65774	134	12170	190477	67848	137
-1.00	- .10	184254	81717	144	28764	190424	66703	134	25747	187074	66931	138
1.00	- .50	184254	81717	144	13960	188813	75030	140	12163	185927	75040	144

Table 10. Results in which the Additional Constraints at the Project Level Had No Effect on the Difference between the Portfolios

		Superordinate Portfolio			Subordinate Portfolios: Branch Constraints Only Imposed				Subordinate Portfolios: Branch and Project Constraints Imposed			
k_1	k_2	Mean	Variance	No. Funded	Sum of Abs. Dif.	Mean	Variance	No. Funded	Sum of Abs. Dif.	Mean	Variance	No. Funded
1.00	- .10	182663	48549	120	45548	190772	66768	137	45449	187430	67765	135
.50	- .10	190960	58319	129	21949	191084	66402	134	21920	189725	67508	137
.50	- .50	190960	58319	129	32200	189615	73631	141	32200	189615	73631	140
- .50	-1.00	189952	74705	139	13316	181744	79237	146	13316	181744	79237	147
-1.00	1.00	185254	82138	144	60197	180969	52414	124	60197	180969	52414	126
-1.00	.50	184254	81717	144	46177	186760	58052	129	46177	286760	53052	131
-1.00	.10	184254	81717	144	35408	190013	63962	131	35408	190013	63962	134

Table 11. Results in which the Additional Constraints at the Project Level Caused the Difference between the Portfolios to Increase

					Subordinate Portfolios							
		Superordinate Portfolio			Branch Constraints				Project Constrainst			
k_1	k_2	Mean	Variance	No. Funded	Sum of Abs. Dif.	Mean	Variance	No. Funded	Sum of Abs. Dif.	Mean	Variance	No. Funded
1.00	.50	182663	48549	120	21366	189527	57044	129	25093	185403	58014	130
1.00	.10	182663	48549	120	38406	190952	62992	132	40365	187713	61981	132
1.00	- .50	182663	48549	120	54952	188632	73339	139	56417	184873	72851	143
1.00	-1.00	182663	48549	120	60430	183775	79478	146	61451	182871	77691	148
.50	.10	190960	58319	129	12328	191435	62570	132	14875	187954	64612	135
.50	-1.00	190960	58319	129	42476	184131	79752	145	43568	181233	78273	148
.10	- .50	191724	63513	130	18214	189433	74585	140	19528	185476	73350	138
- .10	.50	192087	67254	133	21220	189365	58214	130	22090	188260	58747	131
- .10	- .50	192087	67254	133	14208	188236	74053	142	14768	186799	73822	140
- .50	.50	189952	74705	139	32940	189384	58543	129	34124	188233	53477	134
- .50	.10	189952	74705	139	18856	191256	64177	131	19788	188626	63508	134

folios would be classified. The measure used was the sum of absolute differences. If the sum of absolute differences decreased by 200 or more, then the additional constraints were said to be effective. As can be seen in Table 9, the sum of absolute differences decreased from a minimum of 239 to a maximum of 4376. The additional constraints were considered to be ineffective if they produced a change of less than 200 in the sum of absolute differences. In Table 10, it can be observed that the minimum change was 0 and the maximum was 99. To be classified as increasing the difference in the portfolios, the imposition of additional constraints had to increase the sum of absolute differences by at least 200. The changes in this measure ranged from a minimum of 560 to a maximum of 3727 as can be seen in Table 11. More specifically, there were 12 cases in which the additional constraints had the effect of moving the portfolios into closer agreement, 7 cases in which these constraints had no effect, and 11 cases in which the constraints had the effect of decreasing the agreement between the portfolios.

These results were quite unexpected; however, after careful consideration, it becomes logical for the model to behave in the observed manner. As discussed earlier in the chapter, each unit within the subordinate for which the superordinate communicates a budget becomes a separate decision problem. If branch budgets are communicated, then each branch must be funded separately; if project budgets are communicated, then each project must be funded individually. The funding of each branch or project becomes an independent decision problem. The behavior possible when these budgets are specified by the superordinate is discussed in detail earlier in the chapter. Within each project, there will either be additional tasks funded and/or tasks will have their funding levels increased or fewer tasks will be funded and/or tasks will have their funding levels reduced. These behaviors can have possible effects on the difference between the portfolios: they can decrease the difference between them; they can increase the difference between them; or they can have no effect on the portfolios. These effects can be noted in a project by project comparison.

The net effect on the difference between the overall portfolios is the sum of the net effects on the individual projects. An example of the effects of imposing project constraints is presented to illustrate the above procedure.

Let us say we have only two projects and five tasks within each project. Let us label these tasks as follows:

task ij

where i = the project to which the task belongs and j indicates the number of that task within the project. Now let us assume that the superordinate level selects the following tasks for his portfolio at the following funding levels:

Branch 1		8000	
	Project 1	5000	
		Task 11	1000
		Task 13	500
		Task 14	2000
		Task 15	1500

Project 2	3000
Task 21	1000
Task 24	2000

Let us now say that the subordinate, acting under only branch constraints, selects and finds a portfolio as follows:

Branch	8000
Project 1	2300
Task 11	800
Task 12	500
Task 15	1000
Project 2	5700
Task 21	1000
Task 22	1200
Task 23	1000
Task 24	1100
Task 25	1400

This causes a sum of absolute differences of 7700.

Now let us assume that, under project constraints, the subordinate level selects and funds his portfolio as follows:

Branch	8000
Project 1	5000
Task 11	800
Task 12	500
Task 14	2000
Task 15	1300

Project 2	3000
Task 22	1400
Task 25	1600

This portfolio will have a sum of absolute differences of 7500. Hence, the additional constraints have had the desired effect. However, we could have had a subordinate level that selected and funded the following portfolio under the added project constraints:

Branch	8000
Project 1	5000
Task 11	800
Task 12	500
Task 13	2000
Task 15	1700
Project 2	3000
Task 22	1400
Task 25	1600

This portfolio differs from the superordinate level portfolio with a sum of absolute differences equal to 10,400. Thus, the additional constraints caused the portfolios to become more dissimilar rather than similar.

Tables 12, 13, 14, 15, 16, and 17 present the results for each level of superordinate risk tolerance. These results give no indication under which addition of project budgets to the subordinate level in the multi-level decision process will cause the portfolios to become more similar. The only indication of any sort of trend comes from Tables 9, 10, and 11. Here it is shown that, in 73.5 percent of the cases looked

Table 12. Sum of Absolute Differences and Number of Tasks Funded for $k_1 = 1.00$ and Varying Values of k_2 and for Different Constraint Conditions

$k_1 = 1.00$	Branch Constraints		Project Constraints	
k_2	Sum Abs. Differences	No. Tasks Funded	Sum Abs. Differences	No. Tasks Funded
.50	21366	129	25093	130
.10	38406	132	40365	132
- .10	45548	137	45449	135
- .50	54952	139	56417	143
-1.00	60430	146	61451	148
	Superordinate Funds 120			

Table 13. Sum of Absolute Differences and Number of Tasks Funded for $k_1 = 0.50$ and Varying Values of k_2 and for Different Constraint Conditions

$k_1 = .50$	Branch Constraints		Project Constraints	
k_2	Sum Abs. Differences	No. Tasks Funded	Sum Abs. Differences	No. Tasks Funded
1.00	24620	123	19482	127
.10	12328	132	14875	135
- .10	21949	134	21920	137
- .50	32200	141	32200	140
-1.00	42476	145	43568	148
	Superordinate Funds 129			

Table 14. Sum of Absolute Differences and Number of Tasks Funded for $k_1 = 0.10$ and Varying Values of k_2 and for Different Constraint Conditions

$k_1 = .10$	Branch Constraints		Project Constraints	
k_2	Sum Abs. Differences	No Tasks Funded	Sum Abs. Differences	No. Tasks Funded
1.00	38844	122	30928	124
.50	14988	128	11428	129
- .10	6320	124	4768	133
- .50	18214	140	19528	138
-1.00	33246	146	31240	145
	Superordinate Funds 130			

Table 15. Sum of Absolute Differences and Number of Tasks Funded for $k_1 = -0.10$ and Varying Values of k_2 and for Different Constraint Conditions

$k_1 = -.10$	Branch Constraints		Project Constraints	
k_2	Sum Abs. Differences	No. Tasks Funded	Sum Abs. Differences	No. Tasks Funded
1.00	44346	122	38846	128
.50	21220	130	22090	131
.10	6660	131	6000	133
- .50	14208	142	14768	140
-1.00	26074	143	25546	149
	Superordinate Funds 133			

Table 16. Sum of Absolute Differences and Number of Tasks Funded for $k_1 = -0.50$ and Varying Values of k_2 and for Different Constraint Conditions

$k_1 = -.50$	Branch Constraints		Project Constraints	
k_2	Sum Abs. Differences	No. Tasks Funded	Sum Abs. Differences	No. Tasks Funded
1.00	55168	122	48626	128
.50	32940	129	34124	134
.10	18856	131	19788	134
- .10	15452	134	12170	137
-1.00	13316	146	13316	147

Superordinate Funds 139

Table 17. Sum of Absolute Differences and Number of Tasks Funded for $k_1 = -1.00$ and Varying Values of k_2 and for Different Constraint Conditions

$k_1 = -1.00$	Branch Constraints		Project Constraints	
k_2	Sum Abs. Differences	No. Tasks Funded	Sum Abs. Differences	No. Tasks Funded
1.00	60197	124	60197	126
.50	46177	129	46177	131
.10	35408	131	35408	134
- .10	28764	134	25747	138
- .50	13960	140	12163	144

Superordinate Funds 144

at in the superordinate level, risk tolerance was negative. The addition of project budgets caused the portfolios to become more alike or did not cause them to become more dissimilar. In 66.7 percent of the cases in which the superordinate level risk tolerance was positive, the addition of project budgets caused the portfolios to become more dissimilar or not to become more similar. The reason for this behavior is an excellent question which must still be answered.

Summary

This chapter presents the four conjectures tested and the results of these tests. Two of these conjectures deal with single level decision processes and two with multi-level decision processes. The most important result obtained was that the imposition of additional constraints did not necessarily cause the portfolios to become more similar. This tentative result along with the other tentative conclusions, is an attempt to gain insight into multi-level decision processes. Further insights are still to be found and the emphasis of future research should be to gain further insights and to investigate the tentative conclusions presented in this work in much greater detail.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The purpose of this research was to examine some particular aspects of the multi-level decision process in an R & D environment. To this end, four specific conjectures were examined. Through the examination of these conjectures, certain tentative insights were reached.

These insights can be classified into two categories, those dealing with single-level decision processes and those dealing with multi-level decision processes. The single-level decision process insights are:

1. The number of tasks funded by a decision maker is a function of that decision maker's risk tolerance. It was further found that the more risk tolerant a decision maker, the greater would be the number of tasks he selected for funding.

2. The expected value (mean) of the selected portfolio was not symmetrical about k=0. The reasons for this asymmetric behavior are not readily apparent and need to be investigated in greater detail.

The last two tentative insights concern multi-level decision processes. These are fascinating and while the trend appears in this work, certainly more detailed and extensive investigation of these trends is needed. Specifically, more detailed knowledge is necessary about conjecture 4, the most unexpected result achieved in this research. The tentative multi-level insights derived are:

3. The difference between the superordinate level's portfolio and the portfolio selected by a multi-level process is symmetrical about $|k_1 - k_2| = 0$.

4. The imposition of additional constraints upon the subordinate level by the superordinate level will not necessarily cause the superordinate level's portfolio and the superordinate-subordinate level's portfolio to become more alike.

Recommendations

This research has barely begun to scratch the surface of a very large and complex area. It has identified some tentative insights in this area and also indicated other areas where further research is called for. The work presented in this paper is fundamentally of an exploratory nature. Some of the specific areas in which further work is necessary are:

1. The effects of factors other than risk tolerance upon the multi-level decision process.

2. More detailed investigation into the asymmetry property indicated in conjecture 2.

3. Further work on the composition of additional (other than branch) constraints from superordinate to subordinate in order to achieve similarity in portfolio selection; the precise conditions under which this strategy will be successful and when it will not need to be identified.

4. Work in this same area making different assumptions about the data, or using real life data to see if the data have biased any of the results.

Hence, one can see that there is a large variety of problems within this research area which calls for intensive investigation. It is only when all these investigations are completed that we will be able to answer some of the questions which this and other works are now beginning to raise.

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